# RELATIVE FREQUENCIES AND PROBABILITIES

## *Probability*

**STUDY NOTES**

1 **Outcomes** of an event are the possible results. For example, if a coin is tossed there are two possible outcomes, a head or a tail.

2 Probability often deals with **simple experiments** or **trials**.

3 **Random events** occur completely by chance. There is no system, method or pattern if an event occurs **at random**.

4 The **frequency** of an event is the number of times that event occurs. The **relative frequency** is given by $\dfrac{\text{frequency of the event}}{\text{total frequency}}$. For example, if a spinner is spun 30 times and 2 appears 7 times, then the relative frequency of 2 is $\dfrac{7}{30}$.

5 **Relative frequency** can be used to **predict future experimental outcomes**. For example, if the same spinner was spun 120 times we would expect 2 to appear 28 times ($\dfrac{7}{30} \times 120$). Using a large number of trials gives a more accurate relative frequency, which in turn is more efficient at future predictions.

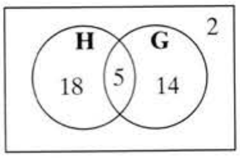
6 ***P*(*A*)** is the **probability of an event** $A$. $P(A) = \dfrac{\text{number of outcomes that give } A}{\text{total number of possible outcomes}}$

For example, when a die is tossed there are 6 possible outcomes (1, 2, 3, 4, 5 or 6). There is one chance in 6 of tossing a 2. We say the probability of tossing a 2 is $\dfrac{1}{6}$.

7 If an event, $A$, is **impossible** then $P(A) = 0$. If the event is **certain**, then $P(A) = 1$. Otherwise the probability of an event is always between 0 and 1. $\mathbf{0 \le P(A) \le 1}$

8 **Lists and tables** can be used to keep track of the sample space. For example, a table is the best way to consider the results when two dice are thrown together. Use a system when creating a list. That way you are less likely to leave things out.

9 A **Venn diagram** is another tool used in probability. Intersecting circles are used to keep track of the number of outcomes favourable to certain events. For example, this Venn diagram has been drawn to show the numbers of students studying either history or geography. A total of 39 (18 + 5 + 14 + 2) students were surveyed. 2 studied neither history nor geography. (They are outside the circles.) 5 studied both subjects. (They lie inside both circles.) 18 studied only history but 23 (18 + 5) studied history. 14 studied only geography but 19 (14 + 5) studied geography altogether.




### Checklist

**Can you:**

1 *Understand the terms 'outcomes', 'event', 'at random', 'probability' and 'relative frequency'?* ☐
2 *Find the relative frequency from the results of experiments?* ☐
3 *Find the probability of simple events?* ☐
4 *Understand that probability will never be less than 0 or greater than 1?* ☐
5 *Understand what it means if the probability of an event is 0 or 1?* ☐
6 *Create tables or lists?* ☐
7 *Understand and use Venn diagrams?* ☐

# TWO-STEP CHANCE EXPERIMENTS

## *Probability*

**STUDY NOTES**

1 In probability we consider the likelihood of certain events. The set of all possible outcomes is called the **sample space**. For example, if the colour of a card chosen at random from a pack of playing cards is noted, the sample space is red and black.

2 $P(A)$ is the **probability of an event $A$**. It is the **chance** that the event $A$ occurs.

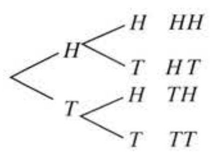
$$P(A) = \frac{\text{number of outcomes that give } A}{\text{number of outcomes in the sample space}}$$

3 The **probability of an event** is usually expressed as a **fraction**, but it could also be given as a decimal or percentage. If the probability of an event is **0**, the event is **impossible**. If the probability of an event is **1**, the event is **certain**. Otherwise the probability of an event will be **between 0 and 1**. The **sum** of the probabilities of **all possible outcomes** must always be **1**.

4 If two events are such that if one doesn't happen then the other must, the events are **complementary.** The probability of an event is one minus the probability of the complementary event. For example, if the probability that it will rain tomorrow is $\frac{1}{3}$, the probability that it will not rain tomorrow is $1 - \frac{1}{3} = \frac{2}{3}$.

5 When considering **two-step events** we need to know if the likelihood of one part occurring is influenced by the likelihood of the other. It is important to know when balls are chosen from a bag, for example, whether it is **with or without replacement**; that is, whether or not the first ball is replaced before the second ball is drawn. The reason why this is important is because the probability for the second ball will be different if the first ball is not replaced.

6 A **tree diagram** is simply a tool used to help determine the **sample space** and find **probabilities**. For example, if drawing a tree diagram to show the possible outcomes if a coin is tossed twice, the first coin could either be a head or a tail. If the first coin was a head, the second coin could either be a head or a tail and if the first coin was a tail, the second coin can either be a head or tail. So, there are four different possible outcomes.




The probability of two heads is $\frac{1}{4}$ and the probability of a head and a tail (in any order) is $\frac{2}{4} = \frac{1}{2}$.

7 In **compound events** if we can find the sample space, we can then determine the **favourable outcomes** and then the probability. For example, if tossing two dice and determining the probability of a sum less than 4, by drawing a table we can find there are 36 possible outcomes of which 3 are favourable (1 and 1, 1 and 2, 2 and 1). So the probability of getting a sum less than 4 is $\frac{3}{36}$ or $\frac{1}{12}$.

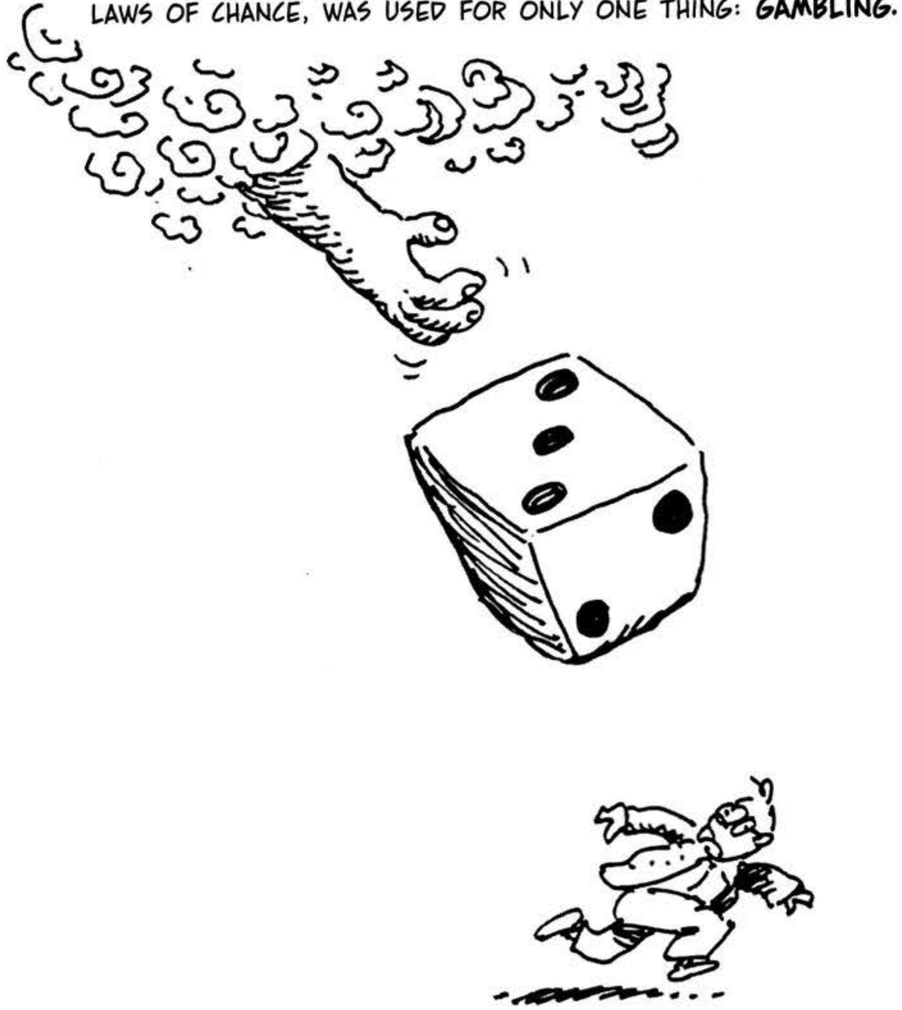
**Checklist**

*Can you:*

1 *Understand what is meant by probability and sample space?* ☐
2 *Find the sample space and probabilities of simple events?* ☐
3 *Understand what is meant by complementary events?* ☐
4 *Understand the need to check whether certain events occur with or without replacement?* ☐
5 *Draw tree diagrams?* ☐
6 *Find probabilities of compound events?* ☐

# Chapter 3
# PROBABILITY

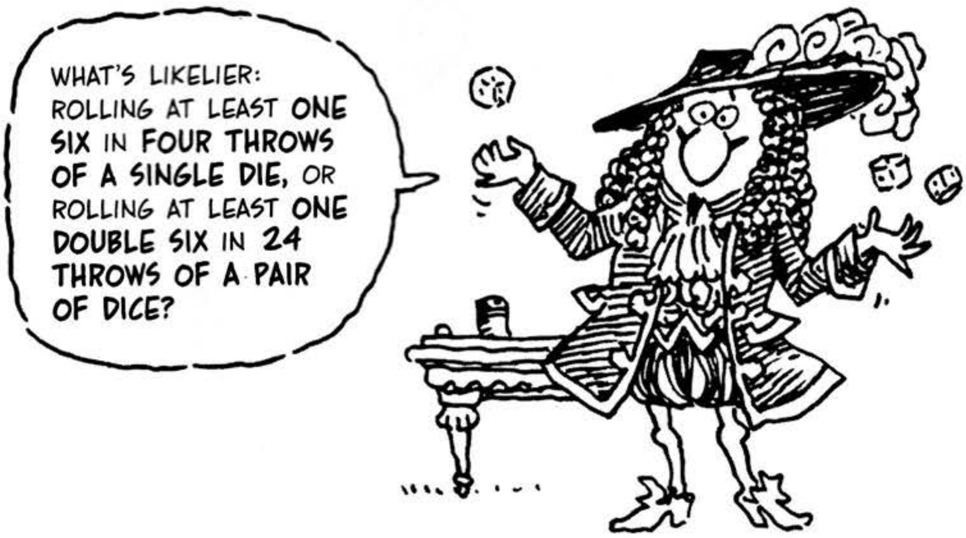
NOTHING IN LIFE IS CERTAIN. IN EVERYTHING WE DO, WE GAUGE THE CHANCES OF SUCCESSFUL OUTCOMES, FROM BUSINESS TO MEDICINE TO THE WEATHER. BUT FOR MOST OF HUMAN HISTORY, **PROBABILITY,** THE FORMAL STUDY OF THE LAWS OF CHANCE, WAS USED FOR ONLY ONE THING: **GAMBLING.**



NOBODY KNOWS WHEN GAMBLING BEGAN. IT GOES AT LEAST AS FAR BACK AS ANCIENT EGYPT, WHERE SPORTING MEN AND WOMEN USED FOUR-SIDED **"ASTRAGALI"** MADE FROM ANIMAL HEELBONES.


THE ROMAN EMPEROR **CLAUDIUS** (10 BCE – 54 CE) WROTE THE FIRST KNOWN TREATISE ON GAMBLING. UNFORTUNATELY, THIS BOOK, "HOW TO WIN AT DICE," WAS LOST.


MODERN DICE GREW POPULAR IN THE MIDDLE AGES, IN TIME FOR A RENAISSANCE RAKE, THE **CHEVALIER DE MERE,** TO POSE A MATHEMATICAL PUZZLER:



THE CHEVALIER REASONED THAT THE AVERAGE NUMBER OF SUCCESSFUL ROLLS WAS THE SAME FOR BOTH GAMBLES:

CHANCE OF ONE SIX $= \frac{1}{6}$

AVERAGE NUMBER IN FOUR ROLLS $= 4(1/6) = \frac{2}{3}$

CHANCE OF DOUBLE SIX IN ONE ROLL $= \frac{1}{36}$

AVERAGE NUMBER IN 24 ROLLS $= 24(1/36) = \frac{2}{3}$

WHY, THEN DID HE **LOSE MORE OFTEN** WITH THE SECOND GAMBLE?


DE MERE PUT THE QUESTION TO HIS FRIEND, THE GENIUS **BLAISE PASCAL** (1623-1666).


ALTHOUGH PASCAL HAD EARLIER GIVEN UP MATHEMATICS AS A FORM OF SEXUAL INDULGENCE (!), HE AGREED TO TACKLE DE MERE'S PROBLEM.

PASCAL WROTE HIS FELLOW GENIUS **PIERRE DE FERMAT,** AND WITHIN A FEW LETTERS, THE TWO HAD WORKED OUT THE THEORY OF PROBABILITY IN ITS MODERN FORM—EXCEPT, OF COURSE FOR THE CARTOONS.

# BASIC DEFINITIONS

AS OUR GAMBLER PLAYS A GAME, WE PLAY SCIENTIST, OBSERVING THE OUTCOME:

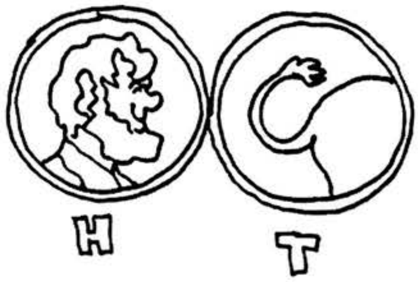
A **random experiment** IS THE PROCESS OF OBSERVING THE OUTCOME OF A CHANCE EVENT.

THE **elementary outcomes** ARE ALL POSSIBLE RESULTS OF THE RANDOM EXPERIMENT.

THE **sample space** IS THE SET OR COLLECTION OF ALL THE ELEMENTARY OUTCOMES.


IF THE EVENT WAS A COIN TOSS, FOR EXAMPLE, THE **RANDOM EXPERIMENT** CONSISTS OF RECORDING ITS OUTCOME.

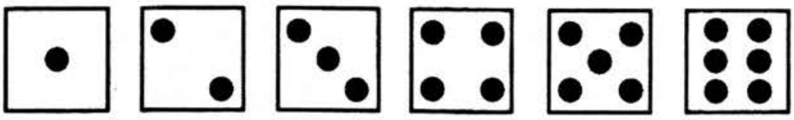
THE **ELEMENTARY OUTCOMES** ARE HEADS AND TAILS.



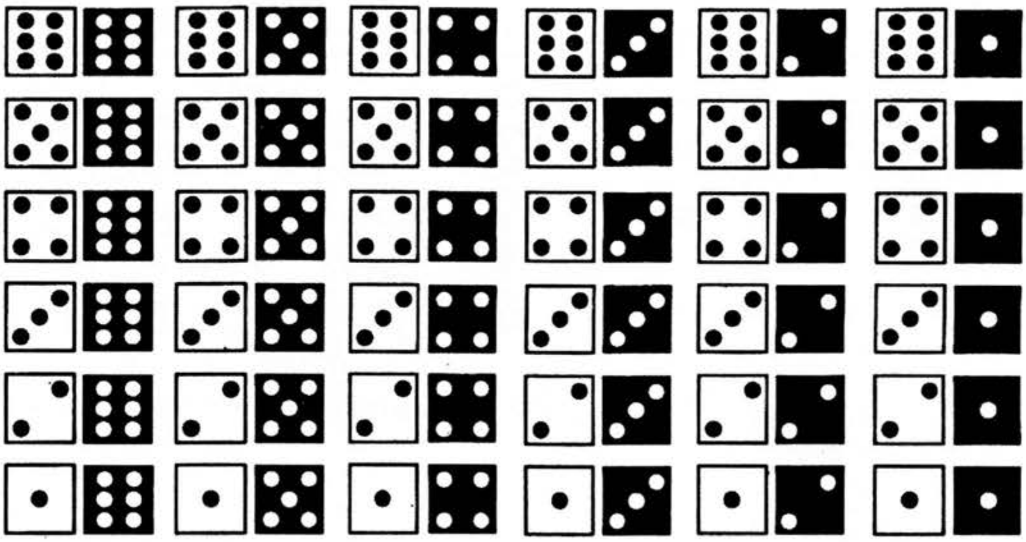

THE **SAMPLE SPACE** IS THE SET WRITTEN

$$\{H, T\}$$

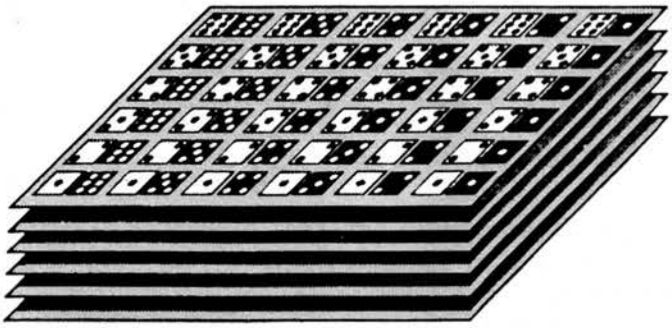
THE SAMPLE SPACE OF THE THROW OF A **SINGLE DIE** IS A LITTLE BIGGER.



AND FOR A **PAIR** OF DICE, THE SAMPLE SPACE LOOKS LIKE THIS (WE MAKE ONE DIE WHITE AND ONE BLACK TO TELL THEM APART):

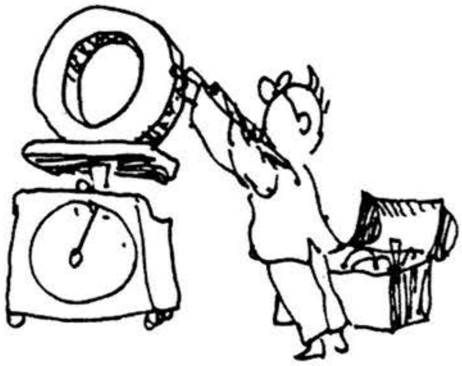


THIS SAMPLE SPACE HAS **36** ($6 \times 6$) ELEMENTARY OUTCOMES. FOR THREE DICE, THE SPACE WOULD HAVE 216 ENTRIES, AS IN THIS $6 \times 6 \times 6$ STACK. AND **FOUR** DICE?




AT SOME POINT, WE HAVE TO STOP LISTING AND START THINKING...

NOW LET'S IMAGINE A RANDOM EXPERIMENT WITH $n$ ELEMENTARY OUTCOMES $O_1$, $O_2$, ... $O_n$. WE WANT TO ASSIGN EACH OUTCOME A **NUMERICAL WEIGHT,** OR **PROBABILITY,** WHICH MEASURES THE LIKELIHOOD OF THAT OUTCOME'S OCCURRING. WE WRITE THE PROBABILITY OF $O_i$ AS $P(O_i)$.



FOR EXAMPLE, IN A FAIR COIN TOSS, HEADS AND TAILS ARE EQUALLY LIKELY, AND WE ASSIGN THEM BOTH PROBABILITY 0.5.

$$P(H) = P(T) = 0.5$$

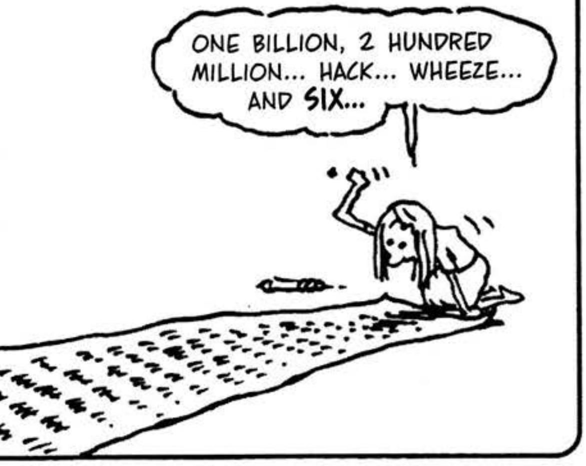
EACH OUTCOME COMES UP HALF THE TIME. ASK ANY **FOOTBALL PLAYER!**


IN THE ROLL OF **TWO** DICE, THERE ARE **36** ELEMENTARY OUTCOMES, ALL EQUALLY LIKELY, SO THE PROBABILITY OF EACH IS 1/36.

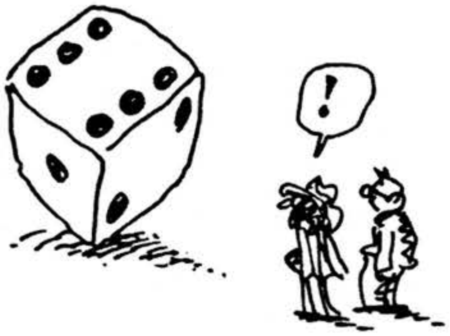
FOR INSTANCE,

$$P(\text{BLACK 5, WHITE 2}) = \frac{1}{36}$$

WHICH MEANS: IF YOU ROLLED THE DICE A VERY LARGE NUMBER OF TIMES, IN THE LONG RUN THIS OUTCOME WOULD OCCUR 1/36 OF THE TIME.

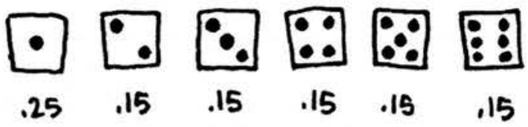


WHAT IF OUR GAMBLER **CHEATS** AND THROWS A **LOADED DIE?** FOR THE SAKE OF ARGUMENT, SUPPOSE THAT NOW A **1** COMES UP **25%** OF THE TIME (IN THE LONG RUN).



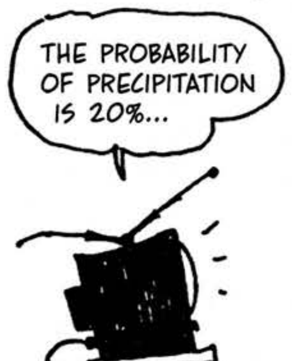

THE **SAMPLE SPACE** IS THE SAME AS FOR A FAIR DIE

$$\{1, 2, 3, 4, 5, 6\}$$




BUT THE PROBABILITIES ARE DIFFERENT. NOW $P(1) = 0.25$ AND THE REMAINING PROBABILITIES ADD UP TO .75. IF 2, 3, 4, 5, AND 6 WERE ALL EQUALLY LIKELY, THEN EACH ONE WOULD HAVE PROBABILITY $0.15 = \frac{1}{5}(.75)$.


IN GENERAL, ELEMENTARY OUTCOMES NEED NOT HAVE EQUAL PROBABILITY.




NOW WHAT CAN WE SAY ABOUT THE PROBABILITIES $P(O_i)$ IN AN ARBITRARY RANDOM EXPERIMENT? FIRST OF ALL,

$$P(O_i) \geq 0$$

PROBABILITIES ARE **NEVER NEGATIVE.** A PROBABILITY OF ZERO MEANS AN EVENT CAN'T HAPPEN. LESS THAN ZERO WOULD BE MEANINGLESS.


SECOND, IF AN EVENT IS **CERTAIN** TO HAPPEN, WE ASSIGN IT PROBABILITY **1.** (IN THE LONG RUN, THAT'S THE PROPORTION OF TIMES IT WILL OCCUR.)

IN PARTICULAR, THE **TOTAL PROBABILITY OF THE SAMPLE SPACE** MUST BE **1.** IF WE DO THE EXPERIMENT, **SOMETHING** IS BOUND TO HAPPEN!


PUT THESE TWO TOGETHER, AND YOU HAVE THE **CHARACTERISTIC PROPERTIES OF PROBABILITY:**

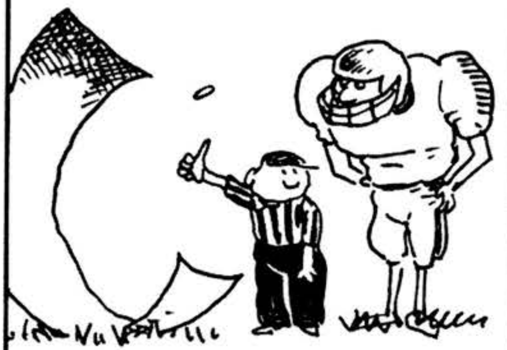
| | |
|---|---|
| $P(O_i) \geq 0$ | PROBABILITY IS **NON-NEGATIVE.** |
| $P(O_1) + P(O_2) + \ldots + P(O_n) = 1$ | TOTAL PROBABILITY OF ALL ELEMENTARY OUTCOMES IS **ONE.** |

LIKE A CLEVER POLITICIAN, WE HAVE AVOIDED CERTAIN **UNPLEASANT QUESTIONS,** SUCH AS A) WHAT DOES PROBABILITY **MEAN?** AND B) **HOW** DO WE ASSIGN PROBABILITIES TO ELEMENTARY OUTCOMES?


HERE ARE SOME APPROACHES THAT HAVE BEEN TAKEN:

**Classical** PROBABILITY: BASED ON GAMBLING IDEAS, THE FUNDAMENTAL ASSUMPTION IS THAT THE GAME IS FAIR AND ALL ELEMENTARY OUTCOMES HAVE THE SAME PROBABILITY.


**Relative Frequency:** WHEN AN EXPERIMENT **CAN** BE REPEATED, THEN AN EVENT'S PROBABILITY IS THE PROPORTION OF TIMES THE EVENT OCCURS IN THE LONG RUN.



**Personal** PROBABILITY: MOST OF LIFE'S EVENTS ARE **NOT REPEATABLE.** PERSONAL PROBABILITY IS AN **INDIVIDUAL'S PERSONAL ASSESSMENT** OF AN OUTCOMES' LIKELIHOOD. IF A GAMBLER BELIEVES THAT A HORSE HAS MORE THAN A 50% CHANCE OF WINNING, HE'LL TAKE AN EVEN BET ON THAT HORSE.


AN **OBJECTIVIST** USES EITHER THE CLASSICAL OR FREQUENCY DEFINITION OF PROBABILITY. A **SUBJECTIVIST** OR **BAYESIAN** APPLIES FORMAL LAWS OF CHANCE TO HIS OWN, OR YOUR, PERSONAL PROBABILITIES.

# BASIC OPERATIONS

SO FAR, WE HAVE DISCUSSED ONLY THE PROBABILITY OF ELEMENTARY OUTCOMES. IN THEORY, THAT WOULD BE ENOUGH TO DESCRIBE ANY RANDOM EXPERIMENT, BUT IN PRACTICE IT'S PRETTY UNWIELDY. FOR EXAMPLE, EVEN SUCH AN ORDINARY OCCURRENCE AS ROLLING A SEVEN IS NOT AN ELEMENTARY OUTCOME... SO WE INTRODUCE A **NEW IDEA.**

AN **EVENT** IS A SET OF ELEMENTARY OUTCOMES. THE PROBABILITY OF AN EVENT IS THE SUM OF THE PROBABILITIES OF THE ELEMENTARY OUTCOMES IN THE SET. FOR INSTANCE, SOME EVENTS IN THE LIFE OF A TWO-DICE ROLLER ARE:

| EVENT DESCRIPTION | EVENT'S ELEMENTARY OUTCOMES | PROBABILITY |
|---|---|---|
| **A:** DICE ADD TO 3 | $\{(1,2),\ (2,1)\}$ | $P(A) = \frac{2}{36}$ |
| **B:** DICE ADD TO 6 | $\{(1,5), (2,4), (3,3), (4,2), (5,1)\}$ | $P(B) = \frac{5}{36}$ |
| **C:** WHITE DIE SHOWS 1 | $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\}$ | $P(C) = \frac{6}{36}$ |
| **D:** BLACK DIE SHOWS 1 | $\{(1,1), (2,1), (3,1), (4,1), (5,1), (6,1)\}$ | $P(D) = \frac{6}{36}$ |

THE BEAUTY OF USING EVENTS, RATHER THAN ELEMENTARY OUTCOMES, IS THAT WE CAN **COMBINE** EVENTS TO MAKE OTHER EVENTS, USING LOGICAL OPERATIONS. THE RELEVANT WORDS ARE **AND, OR,** AND **NOT.**


THAT IS, GIVEN EVENTS *E* AND *F*, WE CAN MAKE NEW EVENTS:

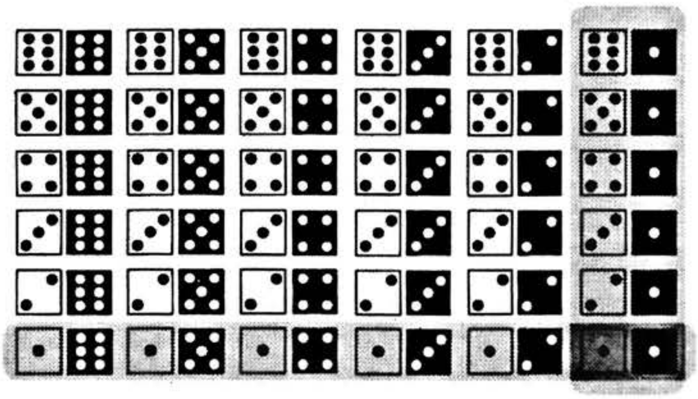
*E* **and** *F*: THE EVENT *E* AND THE EVENT *F* BOTH OCCUR.

*E* **or** *F*: THE EVENT *E* OR THE EVENT *F* OCCURS, OR BOTH DO.

**not** *E*: THE EVENT *E* DOES NOT OCCUR.

COMBINING OUR PRIMITIVE DEFINITIONS OF PROBABILITY WITH THESE LOGICAL OPERATIONS WILL GIVE US SOME POWERFUL FORMULAS FOR MANIPULATING PROBABILITIES.

LET'S RETURN TO THE DICE-THROWING EXAMPLE. IF *C* IS THE EVENT [WHITE DIE = 1], AND *D* IS THE EVENT [BLACK DIE = 1], THEN



***C* OR *D*** IS THE ENTIRE SHADED AREA WHERE ONE DIE OR THE OTHER IS 1.

***C* AND *D*** IS WHERE THE SHADED AREAS OVERLAP, WHERE BOTH DICE ARE 1.

THIS ILLUSTRATES THE ADDITION RULE: FOR ANY EVENTS *E*, *F*,

$$\mathbf{P(E \text{ OR } F) = P(E) + P(F) - P(E \text{ AND } F)}$$

ADDING $P(E)+P(F)$ **DOUBLE COUNTS** THE ELEMENTARY OUTCOMES SHARED BY *E* AND *F*, SO WE MUST SUBTRACT THE EXTRA AMOUNT, WHICH IS ***P*(*E* AND *F*)**.

IN THE ABOVE EXAMPLE,

$$P(C \text{ OR } D) = \frac{11}{36}$$

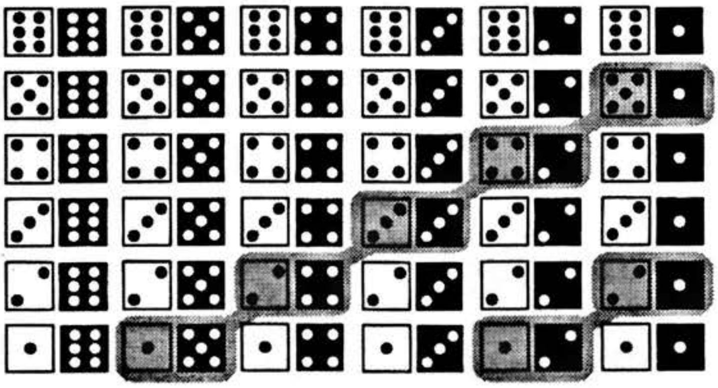
AS YOU CAN SEE BY COUNTING ELEMENTARY OUTCOMES. LIKEWISE,

$$P(C \text{ AND } D) = \frac{1}{36}$$

AND WE CONFIRM THE FORMULA:

$$P(C) + P(D) - P(C \text{ AND } D)$$
$$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$$
$$= P(C \text{ OR } D)$$

SOMETIMES THE OVERLAP ***E* AND *F*** IS EMPTY, AND THE TWO EVENTS SHARE NO ELEMENTARY OUTCOMES. IN THAT CASE, WE SAY *E* AND *F* ARE **MUTUALLY EXCLUSIVE,** MAKING $P(E \text{ AND } F) = 0$. HERE WE SEE THE MUTUALLY EXCLUSIVE EVENTS *A*, THE DICE ADD TO 3, AND *B*, THE DICE ADD TO 6.



FOR MUTUALLY EXCLUSIVE EVENTS, WE GET A SPECIAL ADDITION RULE: IF *E* AND *F* ARE MUTUALLY EXCLUSIVE, THEN

$$P(E \text{ OR } F) = P(E) + P(F)$$

AND WE CHECK THAT $P(A \text{ OR } B) = \frac{7}{36} = \frac{2}{36} + \frac{5}{36} = P(A) + P(B)$

AND FINALLY, A **SUBTRACTION RULE:** FOR ANY EVENT *E*,

$$P(E) = 1 - P(\text{NOT } E)$$

THIS IS USEFUL WHEN $P(\text{NOT } E)$ IS EASIER TO COMPUTE THAN $P(E)$. FOR INSTANCE, LET *E* BE THE EVENT [A DOUBLE-1 IS **NOT THROWN**]. THE EVENT **NOT-*E*,** [A DOUBLE-1 **IS** THROWN], HAS PROBABILITY $P(\text{NOT } E) = 1/36$.

SO

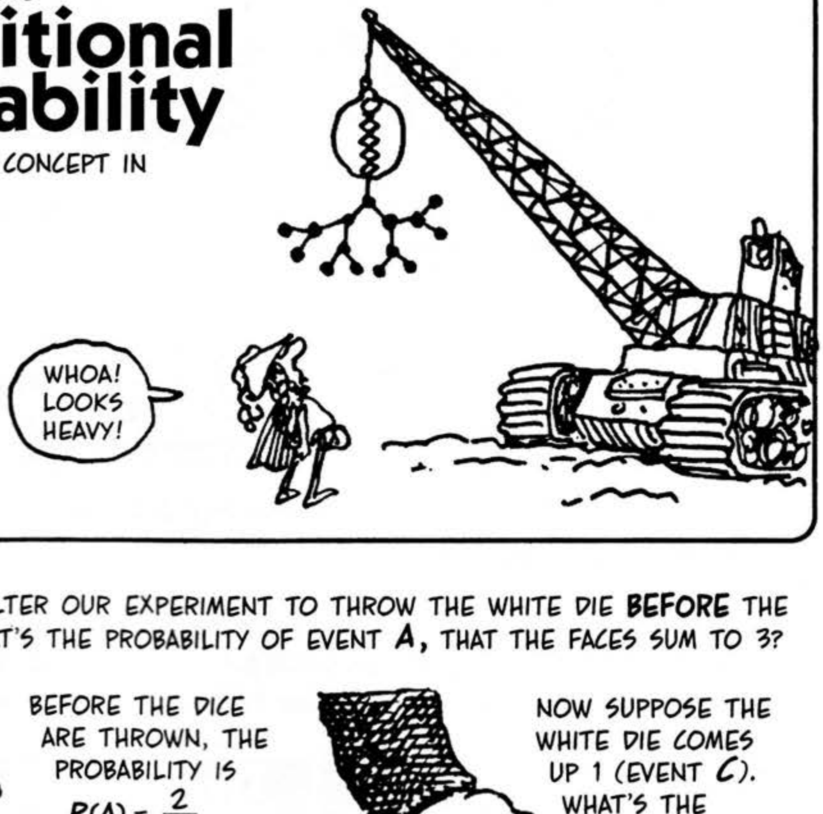
$$P(E) = 1 - P(\text{NOT } E) = 1 - \frac{1}{36} = \frac{35}{36}$$


THE FORMULAS WE JUST DERIVED ARE, IN FACT, ADEQUATE FOR ANSWERING DE MERE'S QUESTION—BUT NOT EASILY! (YOU MIGHT TRY USING THEM ON A SIMPLER QUESTION: WHAT'S THE PROBABILITY OF ROLLING AT LEAST ONE SIX IN TWO ROLLS OF A SINGLE DIE?) WE NEED **MORE MACHINERY!**

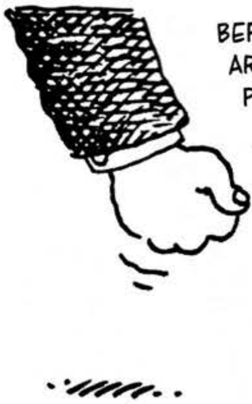
SO WE INTRODUCE

# conditional probability

(AN ESSENTIAL CONCEPT IN STATISTICS!).




SUPPOSE WE ALTER OUR EXPERIMENT TO THROW THE WHITE DIE **BEFORE** THE BLACK DIE. WHAT'S THE PROBABILITY OF EVENT **A**, THAT THE FACES SUM TO 3?



BEFORE THE DICE ARE THROWN, THE PROBABILITY IS

$$P(A) = \frac{2}{36}$$

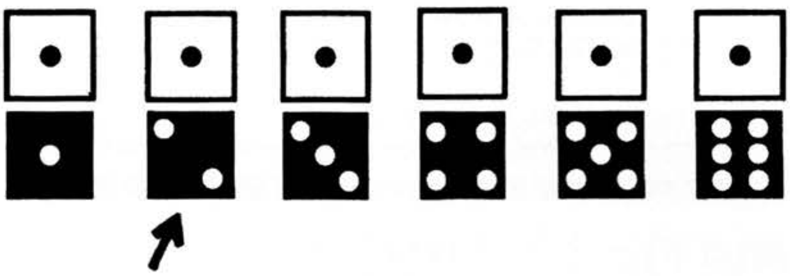
NOW SUPPOSE THE WHITE DIE COMES UP 1 (EVENT $C$). WHAT'S THE PROBABILITY OF **A** NOW?

WE CALL IT THE **CONDITIONAL PROBABILITY** THAT EVENT ***A*** WILL OCCUR, GIVEN THE **CONDITION** THAT EVENT ***C*** HAS ALREADY OCCURRED. WE WRITE

$$P(A|C)$$

AND SAY "THE PROBABILITY OF ***A*, GIVEN *C*.**"


BEFORE ANY DICE WERE THROWN, THE SAMPLE SPACE HAD 36 OUTCOMES, BUT NOW THAT THE EVENT ***C*** HAS OCCURRED, THE OUTCOME MUST BELONG TO THE **REDUCED SAMPLE SPACE *C*.**



IN THE REDUCED SAMPLE SPACE OF SIX ELEMENTARY OUTCOMES, ONLY ONE OUTCOME (1, 2) SUMS TO 3. SO THE CONDITIONAL PROBABILITY IS 1/6.


IN GENERAL, TO FIND THE CONDITIONAL PROBABILITY ***P(E|F)***, WE LOOK AT THE EVENT ***E* AND *F*** AS PART OF THE REDUCED SAMPLE SPACE ***F*.**

WE TRANSLATE THIS INTO A FORMAL DEFINITION: THE **CONDITIONAL PROBABILITY OF $E$, GIVEN $F$,** IS

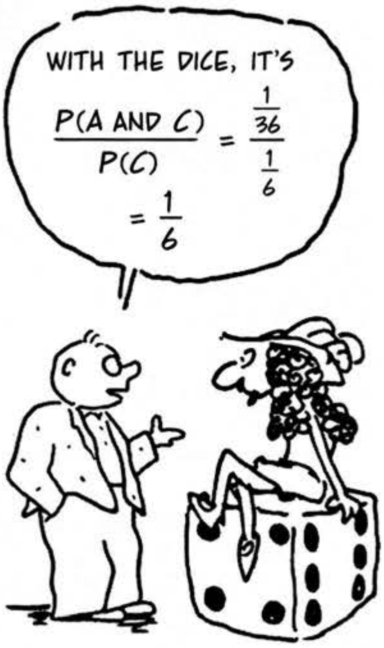
$$P(E|F) = \frac{P(E \text{ and } F)}{P(F)}$$

FROM WHICH YOU CAN DIRECTLY VERIFY SOME INTUITIVE FACTS:

$P(E|E) = 1$ (ONCE $E$ OCCURS, IT'S CERTAIN.)

WHEN E AND F ARE MUTUALLY EXCLUSIVE,

$P(E|F) = 0$ (ONCE $F$ HAS OCCURRED, $E$ IS IMPOSSIBLE.)



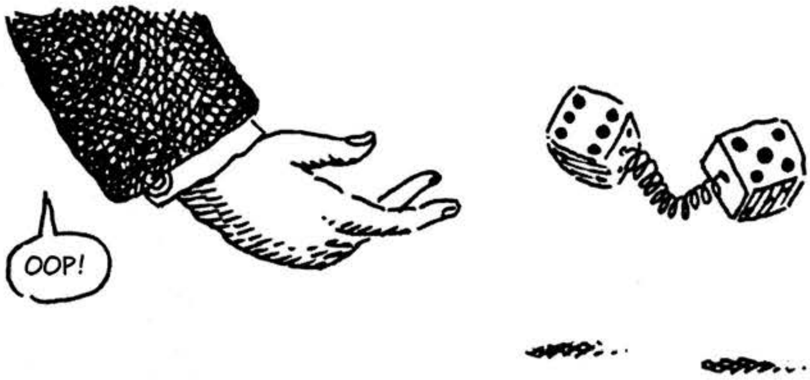

REARRANGING THE DEFINITION GIVES US THE **MULTIPLICATION RULE:**

$$P(E \text{ and } F) = P(E|F)P(F)$$

WHICH WE WOULD LIKE TO REDUCE TO A "SPECIAL" MULTIPLICATION RULE UNDER THE FAVORABLE CIRCUMSTANCES THAT $P(E|F) = P(E)$. THAT WOULD BE EXCELLENT!

# INDEPENDENCE and the special multiplication rule

TWO EVENTS ***E*** AND ***F*** ARE **INDEPENDENT** OF EACH OTHER IF THE OCCURRENCE OF ONE HAS **NO INFLUENCE** ON THE PROBABILITY OF THE OTHER. FOR INSTANCE, THE ROLL OF ONE DIE HAS NO EFFECT ON THE ROLL OF ANOTHER (UNLESS THEY'RE GLUED TOGETHER, MAGNETIC, ETC.!)




IN TERMS OF CONDITIONAL PROBABILITY, THIS AMOUNTS TO SAYING $P(E) = P(E|F)$ OR EQUIVALENTLY, $P(F) = P(F|E)$. WHEN $E$ AND $F$ ARE INDEPENDENT, WE GET A **SPECIAL MULTIPLICATION RULE:**

$$P(E \text{ AND } F) = P(E)P(F)$$

LET'S VERIFY THE INDEPENDENCE OF DICE, USING THE FORMULAS. $C$ IS THE EVENT [WHITE COMES UP 1]; $D$ IS THE EVENT [BLACK DIE COMES UP 1]; AND WE HAVE

$$P(C|D) = \frac{P(C \text{ AND } D)}{P(D)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6} = P(C)$$

BUT THE WHITE DIE SHOWING 1 OBVIOUSLY **DOES** AFFECT THE CHANCES THAT THE SUM OF THE TWO DICE IS 3!

$$P(A|C) = \frac{P(A \text{ AND } C)}{P(C)} = \frac{P(1,2)}{P(C)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6} \neq P(A) = \frac{1}{18}$$

SO THESE TWO EVENTS ARE **NOT INDEPENDENT.**

BEFORE GOING ON, LET'S SUMMARIZE ALL THE RULES WE'VE ACCUMULATED.

ADDITION RULE:

$$P(E \text{ OR } F) = P(E) + P(F) - P(E \text{ AND } F)$$

SPECIAL ADDITION RULE: WHEN $E$ AND $F$ ARE MUTUALLY EXCLUSIVE,

$$P(E \text{ OR } F) = P(E) + P(F)$$

SUBTRACTION RULE:

$$P(E) = 1 - P(\text{NOT } E)$$

MULTIPLICATION RULE:

$$P(E \text{ AND } F) = P(E|F)P(F)$$

SPECIAL MULTIPLICATION RULE: WHEN $E$ AND $F$ ARE INDEPENDENT,

$$P(E \text{ AND } F) = P(E)P(F)$$


AND NOW, DE MERE'S PROBLEM AT LAST... LET $E$ BE THE EVENT OF GETTING AT LEAST ONE SIX IN FOUR ROLLS OF A SINGLE DIE. WHAT'S $P(E)$? THIS IS ONE OF THOSE EVENTS WHOSE NEGATIVE IS EASIER TO DESCRIBE: **NOT $E$** IS THE EVENT OF GETTING **NO SIXES IN FOUR THROWS.**


IF $A_i$ IS THE EVENT [GETTING NO SIX IN THE $i^{TH}$ THROW], WE KNOW THAT $P(A_i) = 5/6$. WE ALSO KNOW THAT ROLLS ARE INDEPENDENT, SO

$$P(\text{NOT } E) =$$

$$P(A_1 \text{ AND } A_2 \text{ AND } A_3 \text{ AND } A_4)$$

MULTIPLICATION RULE

$$= \left(\frac{5}{6}\right)^4 = \mathbf{0.482}$$

SO

$$P(E) = 1 - P(\text{NOT } E) = \mathbf{0.518}$$

NOW THE SECOND HALF: LET $F$ BE THE EVENT OF GETTING AT LEAST ONE DOUBLE SIX IN 24 THROWS. AGAIN, **NOT $F$** IS EASIER TO DESCRIBE. IT'S THE EVENT OF GETTING **NO DOUBLE SIXES.**


IF $B_i$ IS THE EVENT [NO DOUBLE SIX IS THROWN ON THE $i^{TH}$ ROLL], THEN **NOT** $F = B_1$ **AND** $B_2$ **AND**... $B_{24}$. THE PROBABILITY OF EACH $B_i$ IS

$$P(B_i) = \frac{35}{36}, \text{ SO}$$

$$P(\textbf{NOT } F) = \left(\frac{35}{36}\right)^{24} = .509$$

BY THE MULTIPLICATION RULE, AND WE CONCLUDE THAT

$$P(F) = 1 - P(\textbf{NOT } F)$$

$$= 1 - .509 = \mathbf{.491}$$

DE MERE TOLD PASCAL HE HAD ACTUALLY OBSERVED THAT EVENT $F$ OCCURRED LESS OFTEN THAN EVENT $E$, BUT HE WAS AT A LOSS TO EXPLAIN WHY... FROM WHICH WE CONCLUDE THAT DE MERE GAMBLED OFTEN AND **KEPT CAREFUL RECORDS!!**

# RELATIVE FREQUENCIES AND PROBABILITIES

## *Probability*

**SKILLS CHECK**

**1** When a coin was tossed 20 times, a head appeared 12 times. What is the relative frequency of:

a heads? b tails?

**2** A fair die is tossed once. What is the probability that the number on the uppermost face is:

a 4? b 1? c 6? d 7?

**3** A number is chosen at random from the set {1, 2, 3, 4, 5, 6, 7, 8, 9}. What is the probability that the number is:

a 7? b even? c odd?
d less than 10? e 12? f greater than 3?

**4** In an experiment a die was tossed 50 times and the number of times each number appeared was recorded. The results are shown in the table below.

| **Number** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Frequency** | 8 | 7 | 8 | 10 | 9 | 8 |

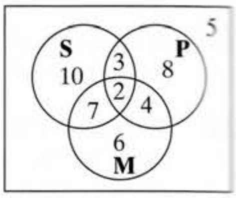
What is the relative frequency of:

a 2? b 5? c 6?

**5** A bag holds 5 black, 4 yellow and 3 green counters. If one counter is selected at random, what is the probability that it is:

a black? b yellow? c red? d not black?

**6** A number of people were surveyed as to whether they played sport, went to a party or went to a movie on a particular weekend. The results are shown in the Venn diagram. How many people:




a were surveyed?
b did not participate in any of these activities over the weekend?
c did all three?
d went to both a movie and a party but didn't play sport?
e played sport?

**7** The table shows the results when a number of people were surveyed about their weight.

| | **Overweight** | **Not overweight** | **Total** |
|---|---|---|---|
| **Men** | 73 | 56 | 129 |
| **Women** | 42 | 60 | 102 |
| **Total** | 115 | 116 | 231 |

a How many people were surveyed?
b What fraction of men were overweight?
c What fraction of women were not overweight?
d What fraction of overweight people were men?

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**Answers** 1 a $\frac{3}{5}$ b $\frac{2}{5}$ 2 a $\frac{1}{6}$ b $\frac{1}{6}$ c $\frac{1}{6}$ d 0 3 a $\frac{1}{9}$ b $\frac{4}{9}$ c $\frac{5}{9}$ d 1 e 0 f $\frac{2}{3}$ 4 a $\frac{7}{50}$ b $\frac{9}{50}$ c $\frac{4}{25}$ 5 a $\frac{5}{12}$ b $\frac{1}{3}$ c 0 d $\frac{7}{12}$
6 a 45 b 5 c 2 d 4 e 22 7 a 231 b $\frac{73}{129}$ c $\frac{10}{17}$ d $\frac{73}{115}$

# RELATIVE FREQUENCIES AND PROBABILITIES

*Probability*

20 MINUTES

**INTERMEDIATE TEST**

## Part A Multiple Choice

**1** A die is thrown once. What is the probability that the number on the uppermost face is 5?

A $\frac{1}{5}$ B $\frac{1}{6}$ C $\frac{5}{6}$ D none of these (1 mark)

**2** A bag holds 5 green and 3 yellow marbles. If a marble is chosen at random, what is the probability that it is yellow?

A $\frac{3}{5}$ B $\frac{3}{8}$ C $\frac{5}{8}$ D none of these (1 mark)

**3** As a car approached each intersection, a passenger recorded the colour of the traffic lights. She found the lights were red 37 times, green 28 times and amber 9 times. What is the relative frequency of amber lights?

A $\frac{9}{28}$ B $\frac{9}{37}$ C $\frac{9}{65}$ D $\frac{9}{74}$ (1 mark)

## Part B Short Answer

**4** A table has been drawn to show the responses when a group of men and women were asked which brand of coffee (X or Y) and which brand of chocolate (A or B) they preferred.

| | X | Y | Total |
|---|---|---|---|
| **A** | 32 | ● | 63 |
| **B** | 19 | 56 | 75 |
| **Total** | 51 | 87 | |

a The number of people who preferred brand A chocolate and brand Y coffee can't be read. What should it be? Hint 1 (1 mark)

b What is the probability that a person:

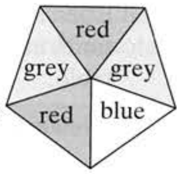
i who preferred brand A chocolate also preferred brand X coffee? (1 mark)

ii who preferred brand Y coffee also preferred brand B chocolate? (1 mark)

iii chosen at random from the group preferred brand B chocolate and brand X coffee? (2 marks)

**5** What is the probability of getting a blue peg when choosing a peg at random from a bag holding 2 blue, 1 green and 3 red pegs? (2 marks)

**6** A cardboard spinner is pentagonal in shape. As shown in the diagram it has two red spaces, two grey spaces and one blue space.




a If the spinner has an equal chance of landing on any of the spaces, what is the probability that it will land on grey? (1 mark)

b The spinner is spun 100 times. It lands on red 37 times and on grey 41 times. What is the relative frequency of blue? Hint 2 (2 marks)

c The spinner was spun 200 times. Using the relative frequency in part **b**, find the number of times it should land on red? Justify your answer. (2 marks)

**7** A survey about pets was taken of 60 households. 38 had at least one dog, 32 had a cat and 14 had both. What is the probability that a household selected at random from this group had neither a cat nor a dog? Hint 3 (2 marks)

**8** A drawing pin was dropped 100 times and a record was made according to whether it landed point down or point up. The results are shown in the table.

| Point down | Point up |
|---|---|
| 45 | 55 |

a What is the relative frequency, as a percentage, of the pin landing point down? (1 mark)

b If the pin was dropped 120 times, approximately how many times would it land point up? Hint 4 (2 marks)

*Hint 1:* Use either row A total or column Y total. Both give the same answer.
*Hint 2:* First find how many times the spinner must have landed on blue.
*Hint 3:* Use a Venn diagram.
*Hint 4:* Use the relative frequency of the pin landing face up.

**Your Feedback**

$\frac{\square}{20} \times 100\% = \square\,\%$

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# RELATIVE FREQUENCIES AND PROBABILITIES

## *Probability*

30 MINUTES

**ADVANCED TEST**

**1** Ellen rolled a die 60 times. The number of times each score was obtained is shown in the table.

| **Number** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Frequency** | 8 | 11 | 10 | 9 | 9 | 13 |

a What is the relative frequency, as a fraction in simplest form, of a score:

i of 2? (1 mark)

ii of 5? (1 mark)

iii that is even? (1 mark)

iv greater than 2? (1 mark)

v less than 3? (1 mark)

b How many times would you expect each score to occur? (1 mark)

c Which scores occurred more often than you would expect? (1 mark)

d A second trial is to be conducted, rolling the die 240 times. Ellen commented that she would expect to get a score of one 32 times. Do you agree? Justify your answer. (2 marks)

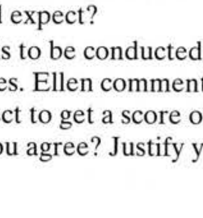
**2** A coin is tossed 24 times. The relative frequency of heads is $\frac{3}{8}$.

a How many times did tails appear? (1 mark)

b In a second trial the same coin was tossed 40 times. The number of tails was the same as in the first trial. What is the relative frequency of heads for the second trial? (1 mark)

c Jonah commented that the overall relative frequency of heads must be $\frac{1}{2}$. Do you agree? Justify your answer. (2 marks)

**3** Elsie made this spinner.




a What is the theoretical probability of spinning 3? (1 mark)

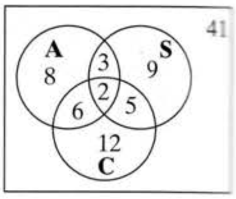
b In three different trials, each involving 36 spins, the number 3 was spun 8, 6 and 7 times respectively. Is this what you would expect? Comment. (2 marks)

c In another trial 2 occurred 8 times. The relative frequency of 2 and 3 respectively was $\frac{1}{3}$ and $\frac{1}{4}$. How many times did 1 occur? (2 marks)

**4** a A bag holds marbles in three different colours. 8 marbles are green, 6 are blue and the rest are red. The probability of randomly choosing a blue marble from the bag is $\frac{2}{9}$. What is the probability of choosing a red marble? (2 marks)

b Another bag also has marbles in the same three colours. The probability of choosing a blue marble is $\frac{1}{4}$ and the probability of choosing a red marble is $\frac{2}{5}$. What is the probability of choosing a green marble? (2 marks)

**5** A Venn diagram has been drawn to show the number of students from Year 9 who represented a school in athletics, swimming or cross-country. What is the probability that a randomly chosen student from Year 9 represented the school in:




a swimming? (1 mark)

b all three sports? (1 mark)

c only cross-country? (1 mark)

d athletics and cross-country but not swimming? (1 mark)

e exactly two of the sports? (1 mark)

f swimming or athletics but not both? (1 mark)

**6** A group of 50 students was asked if they had seen two movies. 19 of the students had seen *Rust* and 23 had seen *Flashy*, while 14 had not seen either movie. If a student is chosen at random from the group, what is the probability that he or she had seen both? (2 marks)

**Your Feedback**

$\frac{\square}{30} \times 100\% = \square\ \%$

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a Range for males = 96 – 43
= 53
Range for females = 97 – 45
= 52 ✓
The range is very slightly higher for males than for females, but it is very similar for both groups. ✓ (2 marks)

b Median for males = 71
Median for females = 76 ✓
So the median is 5 years higher for females than for males. ✓ (2 marks)

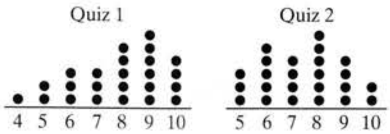
c Mean for males = 70.3 (1 d.p.; by calculator)
Mean for females = 74.8 (1 d.p.; by calculator) ✓
So the mean age for females is 4.5 years older than for males. ✓ (2 marks)

d The modal age for males = 67
The ages for females are bimodal.
The modes for females are 68 and 83. ✓
So the modal age for males is very similar to one of the modal ages for females, but there is a second mode for females that is much higher. As the modal age only occurs once more than most other ages it is not a particularly useful measure. ✓ (2 marks)

e From the stem-and-leaf plot we can immediately see that there are many more males than females. From the mean and median we can also see that males tend to have problems at a younger age than females. ✓ (1 mark)

f The range for males decreases by 9 to 44. ✓
The mean age would decrease to just under 70 (69.7), but the median and mode would not change. ✓
There is now a more obvious difference between the males and females, with the males tending to present at a younger age than females. ✓ (3 marks)

**2**




a There are 24 students in the class. ✓ (1 mark)

b Yes, Jesse is correct. ✓
The range for quiz 1 is 6 while the range for quiz 2 is 5. So the spread of scores is less in quiz 2, (but only slightly). ✓ (2 marks)

c Quiz 1:
mean = 7.8 (1 d.p.)
median = 8 ✓
Quiz 2:
mean = 7.375
median = 7.5 ✓
So, both the mean and median were higher for quiz 1, so that is the quiz in which the students performed best. ✓ (3 marks)

**3** a Total frequency
$= 4 + 5 + 7 + 6 + 2$
$= 24$
There are 24 students in the class. ✓ (1 mark)

b First test:
Mean
$= (4 \times 3 + 5 \times 4 + 7 \times 5 + 6 \times 6 + 2 \times 7) \div 24$
$= 4.875$ ✓
Second test:
Mean
$= (3 \times 4 + 4 \times 5 + 5 \times 6 + 6 \times 7 + 4 \times 8 + 2 \times 9) \div 24$
$= 6.4166...$
Increase
$= 6.4166... - 4.875$
$= 1.541\,666...$
$= 1.5$ (1 d.p.) ✓ (2 marks)

c There are 24 scores altogether. The median in each test is the average of the 12th and 13th score.
First test: Median = 5 ✓
Second test: median = 6.5
Difference = 1.5 ✓ (2 marks)

d First test: range = 7 – 3
= 4
Second test: range = 9 – 4
= 5
The range increased by one mark. ✓ (1 mark)

e On the whole, the students performed better in the second test than in the first test. The mean and median both increased by 1.5. ✓
The most common score in the first test was 5 and in the second test 7, so this also increased. The range was slightly larger for the second test so those marks were a little less consistent. ✓ (2 marks)

**4**

| Number of messages | Frequency in morning | Frequency in afternoon |
|---|---|---|
| 0 | 8 | 5 |
| 1 | 6 | 7 |
| 2 | 1 | 4 |
| 3 | 2 | 3 |
| 4 | 3 | 1 |

Morning: mean = 1.3
median = 1
mode = 0
range = 4 ✓
Afternoon: mean = 1.4
median = 1
mode = 1
range = 4 ✓
The median and range are the same for both times. The mode and mean are slightly higher in the afternoon. ✓
There is not a lot of difference statistically. ✓ (4 marks)

(Total: 30 marks)

RELATIVE FREQUENCIES AND PROBABILITIES

**SKILLS CHECK** PAGE 102

**1** a Relative frequency of heads
$= \frac{12}{20}$
$= \frac{3}{5}$

b Number of tails $= 20 - 12$
$= 8$

Relative frequency of tails
$= \frac{8}{20}$
$= \frac{2}{5}$

**2** a $P(4) = \frac{1}{6}$

b $P(1) = \frac{1}{6}$

c $P(6) = \frac{1}{6}$

d $P(7) = 0$ [It is impossible.]

**3** a There are 9 numbers.
$P(7) = \frac{1}{9}$

b There are 4 even numbers.
$P(\text{even}) = \frac{4}{9}$

c There are 5 odd numbers.
$P(\text{odd}) = \frac{5}{9}$

d All the numbers are less than 10.
$P(\text{number less than 10}) = 1$
[It is certain.]

e $P(12) = 0$ [It is impossible.]

f $P(\text{number greater than 3})$
$= \frac{6}{9}$
$= \frac{2}{3}$

**4** a relative frequency of 2 $= \frac{7}{50}$

b relative frequency of 5 $= \frac{9}{50}$

c relative frequency of 6 $= \frac{8}{50}$
$= \frac{4}{25}$

**5** Total counters $= 5 + 4 + 3$
$= 12$

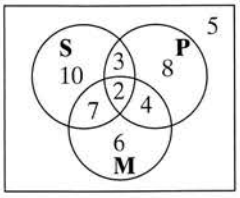
a 5 counters are black.
$P(\text{black}) = \frac{5}{12}$

b 4 counters are yellow.
$P(\text{yellow}) = \frac{4}{12}$
$= \frac{1}{3}$

c No counters are red.
$P(\text{red}) = 0$

d 7 counters are not black.
$P(\text{not black}) = \frac{7}{12}$

**6**




a Number surveyed
$= 10+7+3+2+8+4+6+5$
$= 45$

b Number in no activities = 5
[There are 5 not in any of the circles.]

c Number in all three activities = 2
[There are 2 in all three circles.]

d Number who went to movie and party, no sport = 4

e Number who played sport
$= 10 + 7 + 3 + 2$
$= 22$

**7**

| | Over-weight | Not over-weight | Total |
|---|---|---|---|
| **Men** | 73 | 56 | 129 |
| **Women** | 42 | 60 | 102 |
| **Total** | 115 | 116 | 231 |

a People surveyed = 231

b 73 out of 129 men were overweight; $\frac{73}{129}$

c 60 out of 102 women were not overweight; $\frac{60}{102} = \frac{10}{17}$

d 73 of the 115 overweight people were men; $\frac{73}{115}$

RELATIVE FREQUENCIES AND PROBABILITIES

### INTERMEDIATE TEST PAGE 103

**1** $P(\text{five}) = \frac{1}{6}$ [There are 6 outcomes, one of those six is the desired outcome.]
$\therefore$ [B] ✓ (1 mark)

**2** $P(\text{yellow}) = \frac{3}{8}$
[There are 8 marbles to choose from; 3 of those marbles are yellow.]
$\therefore$ [B] ✓ (1 mark)

**3** Number of traffic lights
$= 37 + 28 + 9$
$= 74$
Number of amber lights = 9
Relative frequency of amber
$= \frac{9}{74}$
$\therefore$ [D] ✓ (1 mark)

**4** a Missing number $= 63 - 32$
$= 31$ ✓
(1 mark)

b i $P(X) = \frac{32}{63}$ ✓ (1 mark)
[32 out of the 63 who prefer brand A]

ii $P(B) = \frac{56}{87}$ ✓ (1 mark)
[56 out of the 87 who prefer brand Y]

iii Total people
$= 63 + 75$
$= 138$ ✓
$P(\text{B and X}) = \frac{19}{138}$ ✓
(2 marks)

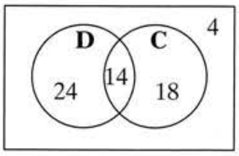
**5** Number of pegs $= 2 + 3 + 1$
$= 6$
$P(\text{blue peg}) = \frac{2}{6}$ ✓
$= \frac{1}{3}$ ✓ (2 marks)

**6** a $P(\text{grey}) = \frac{2}{5}$ ✓ (1 mark)
[There are five spaces, two of which are grey.]

b Lands on red or grey
$37 + 41 = 78$ times
Times it lands on blue
$= 100 - 78$
$= 22$ ✓
Relative frequency of blue
$= \frac{22}{100}$
$= \frac{11}{50}$ ✓ (2 marks)

c Relative frequency of red
$= \frac{37}{100}$ ✓
In 200 spins it should land on red.
$\frac{37}{100} \times 200 = 74$ times. ✓
(2 marks)

**7**




✓

4 households had neither a dog nor a cat.
[14 had both so 38 – 14 = 24 had a dog and not a cat. 32 – 14 = 18 had a cat and not a dog.
24 + 14 + 18 = 56 so 4 (60 – 56) had neither animal.]
$P(\text{neither animal}) = \frac{4}{60}$
$= \frac{1}{15}$ ✓
(2 marks)

**8** a Relative frequency of pin landing point down
$= \frac{45}{100}$
$= 45\%$ ✓ (1 mark)

b Relative frequency of pin landing point up
$= \frac{55}{100}$
$= \frac{11}{20}$ ✓
In 120 trials the pin should land point up approximately
$\frac{11}{20} \times 120 = 66$ times. ✓
[Or: Relative frequency of pin landing point up = 55%. In 120 trials it should land point up 55% of 120 = 66 times.] (2 marks)
(Total: 20 marks)

RELATIVE FREQUENCIES AND PROBABILITIES
**ADVANCED TEST** PAGE 104

**1**

| **Number** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Frequency** | 8 | 11 | 10 | 9 | 9 | 13 |

a i Relative frequency of $2 = \frac{11}{60}$ ✓ (1 mark)

ii Relative frequency of $5 = \frac{9}{60}$
$= \frac{3}{20}$ ✓ (1 mark)

iii Total even scores
$= 11 + 9 + 13$
$= 33$
Relative frequency of even $= \frac{33}{60}$
$= \frac{11}{20}$ ✓ (1 mark)

iv Total greater than 2
$= 10 + 9 + 9 + 13$
$= 41$
Relative frequency
$= \frac{41}{60}$ ✓ (1 mark)

v Total less than 3
$= 8 + 11$
$= 19$
Relative frequency
$= \frac{19}{60}$ ✓ (1 mark)

b $60 \div 6 = 10$
Each score would be expected to occur 10 times.
✓ (1 mark)

c The scores 2 and 6 occurred more times than would be expected. ✓ (1 mark)

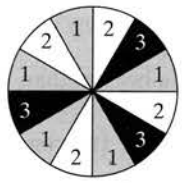
d $240 \div 6 = 40$
Each score would be expected to occur 40 times. ✓
If the relative frequency for 1 was used that would give an expected result of 32, but if the die is fair each score should occur an equal number of times. ✓ (2 marks)

**2** a Relative frequency of heads $= \frac{3}{8}$
So relative frequency of tails $= \frac{5}{8}$
Number of tails
$= \frac{5}{8} \times 24$
$= 15$ ✓ (1 mark)

b Tails occurred 15 times
Number of heads = 40 – 15
= 25
Relative frequency of heads
$= \frac{25}{40}$
$= \frac{5}{8}$ ✓ (1 mark)

c In first trial, number of heads
= 24 – 15
= 9
Total heads = 9 + 25
= 34
Total tosses = 24 + 40
= 64
Relative frequency of heads
$= \frac{34}{64}$
$= \frac{17}{32}$ ✓
The relative frequency is not $\frac{1}{2}$. So, Jonah is wrong.
✓ (2 marks)

**3**




a $P(3) = \frac{3}{12}$
$= \frac{1}{4}$ ✓ (1 mark)

b In 36 spins you would expect to spin 3 a quarter of the times. So you would expect to spin 3 nine times. In three different trials, 3 was spun less than this expected number, although not much less. ✓
It may be that the spinner is not fair, but we need many more trials to be certain of this. ✓ (2 marks)

c 8 is $\frac{1}{3}$ of the number of times spun.
Number of times = 3 × 8
= 24 ✓
Number of times 3 was spun
$= \frac{1}{4} \times 24$
$= 6$

# WORKED SOLUTIONS

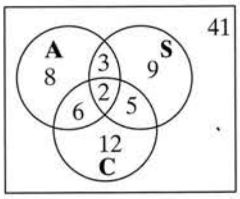
## CHECK YOUR SOLUTIONS

Number of times 1 was spun
$= 24 - (8 + 6)$
$= 10$ ✓ (2 marks)

**4** a There are 6 blue marbles.
$P(\text{blue}) = \frac{2}{9} = \frac{6}{27}$
So there are 27 marbles altogether. ✓
Number of red marbles
$= 27 - (8 + 6)$
$= 13$
$P(\text{red}) = \frac{13}{27}$ ✓ (2 marks)

b $P(\text{blue or red}) = \frac{1}{4} + \frac{2}{5}$
$= \frac{13}{20}$ ✓
$P(\text{green})$
$= 1 - P(\text{blue or red})$
$= 1 - \frac{13}{20}$
$= \frac{7}{20}$ ✓ (2 marks)

**5**




a Total Year 9 students = 86
Number representing in swimming
$= 3 + 9 + 2 + 5$
$= 19$
$P(\text{swimming}) = \frac{19}{86}$ ✓ (1 mark)

b $P(\text{all 3 sports}) = \frac{2}{86}$
$= \frac{1}{43}$ ✓ (1 mark)

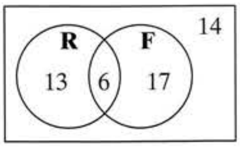
c $P(\text{only cross-country})$
$= \frac{12}{86}$
$= \frac{6}{43}$ ✓ (1 mark)

d $P(\text{athletics, cross-country, no swimming})$
$= \frac{6}{86}$
$= \frac{3}{43}$ ✓ (1 mark)

e Number representing in exactly 2 sports
$= 3 + 5 + 6$
$= 14$
$P(\text{exactly 2 sports})$
$= \frac{14}{86}$
$= \frac{7}{43}$ ✓ (1 mark)

f Number in swimming or athletics but not both
$= 9 + 5 + 6 + 8$
$= 28$
$P(\text{swimming or athletics but not both})$
$= \frac{28}{86}$
$= \frac{14}{43}$ ✓ (1 mark)

**6** 14 had not seen either movie.
Number who had seen a movie
$= 50 - 14$
$= 36$
Now $19 + 23 = 42$
$42 - 36 = 6$
So 6 students must have seen both. ✓




$P(\text{seen both}) = \frac{6}{50}$
$= \frac{3}{25}$ ✓ (2 marks)

(Total: 30 marks)

TWO-STEP CHANCE EXPERIMENTS

**SKILLS CHECK** PAGE 106

**1** a head, tail
b 6, 7, 8, 9

**2** a 1, 2, 3, 4, 5, 6
b A, E, G, N, R, S, T

**3** $P(\text{not k}) = 1 - \frac{1}{26}$
$= \frac{25}{26}$

**4**

| + | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

a $P(8) = \frac{5}{36}$

b $P(\text{less than 4}) = \frac{3}{36}$
$= \frac{1}{12}$

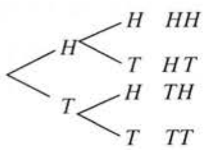
c $P(\text{greater than 6}) = \frac{21}{36}$
$= \frac{7}{12}$

d $P(\text{even}) = \frac{18}{36}$
$= \frac{1}{2}$

**5** a If the first pen is replaced there are 10 pens to choose from, 7 of which are blue.
$P(\text{blue}) = \frac{7}{10}$

b If the first pen is not replaced, 9 pens remain, 6 of which are blue.
$P(\text{blue}) = \frac{6}{9}$
$= \frac{2}{3}$

**6**




a $P(HH) = \frac{1}{4}$

b $P(TT) = \frac{1}{4}$

c $P(\text{one head and one tail})$
$= \frac{2}{4} = \frac{1}{2}$

d $P(\text{at least one head}) = \frac{3}{4}$

**7** a Total tees $= 5 + 4$
$= 9$
$P(\text{white}) = \frac{4}{9}$

# TWO-STEP CHANCE EXPERIMENTS

*Probability*

**SKILLS CHECK**

**1** List all the possible outcomes if:
- a a coin is tossed once
- b a whole number, greater than 5 and less than 10, is chosen at random

**2** List the sample space if:
- a a die is thrown once
- b a letter is chosen at random from the word 'STRANGE'

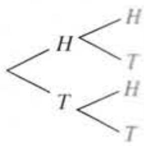
**3** The probability that a letter chosen at random from the alphabet is 'k' is $\frac{1}{26}$. What is the probability that the letter is not 'k'?

**4** The table shows the possible outcomes if two dice are tossed together and the numbers on the uppermost faces added. What is the probability that the total is:
- a 8?
- b less than 4?
- c greater than 6?
- d even?

| + | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

**5** A box holds 7 blue and 3 red pens. Dan takes out a pen and finds that it is blue. He then takes a second pen at random. What is the probability that the second pen is blue if:
- a the first pen is replaced before the second pen is selected?
- b the first pen is not replaced before the second pen is selected?

**6** A tree diagram has been drawn to show the possible results when two coins are tossed. What is the probability that:
- a both coins show heads?
- b both coins show tails?
- c one coin shows a head and one shows a tail?
- d at least one coin shows a head?




**7** A bag holds 5 yellow and 4 white golf tees. One tee is chosen at random from the bag.
- a What is the probability that it is white?
- b A second tee is chosen at random. What is the probability that it is also white (the first tee not having been replaced)?
- c If the first tee was replaced, what is the probability that the second tee chosen is white?

**8** There are 3 marbles in a box; one is red, one blue and one green. Mia takes a marble from the box, notes the colour and then replaces it. She then again takes a marble from the box.
- a Draw a tree diagram to show the possible results.
- b What is the probability that one marble is red and one is blue?

**9** There are 3 marbles in a box; one is red, one blue and one green. Pia takes a marble from the box and does not replace it. She then again takes a marble from the box.
- a Draw a tree diagram to show the possible results.
- b What is the probability that one marble is red and one is blue?

PAGES 187–188

**Answers** 1 a H, T b 6, 7, 8, 9 2 a 1, 2, 3, 4, 5, 6 b A, E, G, N, R, S, T 3 $\frac{25}{26}$ 4 a $\frac{5}{36}$ b $\frac{1}{12}$ c $\frac{7}{12}$ d $\frac{1}{2}$ 5 a $\frac{7}{10}$ b $\frac{2}{3}$ 6 a $\frac{1}{4}$ b $\frac{1}{4}$ c $\frac{1}{2}$ d $\frac{3}{4}$ 7 a $\frac{4}{9}$ b $\frac{3}{8}$ c $\frac{4}{9}$ 8 a see page 188 b $\frac{2}{9}$ 9 a see page 188 b $\frac{1}{3}$

# TWO-STEP CHANCE EXPERIMENTS

*Probability*

20 MINUTES

**INTERMEDIATE TEST**

## Part A Multiple Choice

**1** Which could NOT be the probability of an event? *Hint 1*

A 0 B $\frac{2}{5}$ C $\frac{5}{2}$ D 1 (1 mark)

**2** There are four blue, three red and two green pegs in a bag. Two pegs are taken from the bag, one after the other, without replacement.
The first peg is red. What is the probability that the second peg is also red?

A $\frac{1}{3}$ B $\frac{1}{4}$ C $\frac{2}{9}$ D $\frac{3}{8}$ (1 mark)

**3** Two coins are tossed together. What is the probability that both show tails?

A $\frac{1}{4}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D none of these (1 mark)

**4** There are three ribbons in a box. Two are pink and one is blue. One ribbon is taken from the box, its colour is noted, and then it is replaced. A second ribbon is then taken. What is the probability that both ribbons are blue?

A 0 B $\frac{1}{9}$ C $\frac{1}{3}$ D none of these (1 mark)

## Part B Short Answer

**5** There are four calves in a yard. Two calves are black, one is red and one is white. A buyer is choosing two of the calves. Each of the calves is equally likely to be chosen.

a Draw a tree diagram to show the possible pairings of calves. *Hint 2* (2 marks)

b What is the probability that both chosen calves are black? (1 mark)

c What is the probability that at least one of the chosen calves is red? (1 mark)

d The buyer makes her first choice and chooses one of the black calves. What is the probability that the other calf will also be black? *Hint 3* (1 mark)

**6** Paris and Dane play a game. They throw two dice together and subtract the lower value from the higher to form the score. (The score is zero if the numbers on the dice are the same.) Paris wins the game if the score is 1 or 3. Dane wins if the score is 2, 4 or 5. Who has the greater chance of winning? Justify your answer. *Hint 4* (3 marks)

**7** There are three pens in a case: one black, one blue and one red. Without looking, Julie takes two pens from the case.

a What is the probability that the pens are black and blue? (2 marks)

b James knows that the first pen Julie took was black. What is the probability that the pens are black and blue? (1 mark)

**8** There are five cards in a hat. The letters A, B, C, D and E are written on the cards, one letter on each card. One card is taken, without looking, from the hat. The first card is not replaced and a second card is then drawn.

a What is the probability that the first card shows B? (1 mark)

b What is the probability that the second card shows C? (1 mark)

c Make a list of all the possible outcomes for the two cards. (1 mark)

d What is the probability that the cards are D and E, in any order? (1 mark)

e If the first card was replaced before the second card was drawn, what is the probability that the cards are D and E (in any order)? *Hint 5* (1 mark)

*Hint 1:* Between what two values must all probabilities lie?
*Hint 2:* There is no replacement.
*Hint 3:* The first calf is black. Only look at that part of the tree diagram.
*Hint 4:* Draw a table to show possible outcomes.
*Hint 5:* Add more outcomes to your list.

**Your Feedback**

$\frac{\square}{20} \times 100\% = \square\ \%$

PAGE 188

PAGE 198

# TWO-STEP CHANCE EXPERIMENTS

## *Probability*

30 MINUTES

**ADVANCED TEST**

1 One red, one green and two blue pencils are on a desk.

a Draw a tree diagram and show the sample space if two pencils are taken (without replacement). (2 marks)

b What is the probability that:

i both pencils are blue? (1 mark)

ii both pencils are the same colour? (1 mark)

iii the first pencil is red? (1 mark)

iv exactly one pencil is blue? (1 mark)

v neither pencil is blue? (1 mark)

vi neither is red? (1 mark)

viii one is green and one is blue? (1 mark)

c If the first pencil is replaced before the second is taken will the probability of getting 2 blue pencils be greater or less than before? *Justify your answer.* (2 marks)

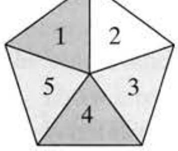
2 There are six mugs on a table, four are white and two are blue. Jack takes one of the mugs and then Cath takes one of the remaining ones.

a What is the probability that Cath's mug is blue? (1 mark)

b Jack's mug was white. What is the probability that Cath's mug is blue? (1 mark)

c Briefly explain the significance of knowing the colour of Jack's mug for the probability of Cath's mug. (1 mark)

3 This spinner is spun twice.




What is the probability that:

a the same number will be spun each time? (1 mark)

b the two numbers add to 6? (1 mark)

c the first number is greater than the second number? (1 mark)

4 A bucket holds 5 green, 3 red and 2 yellow balls. Two balls are taken from the bucket to be used in a game. What is the probability that:

a both balls are red? (1 mark)

b at least one ball is green? (2 marks)

c exactly one ball is green? (1 mark)

5 There are 10 songs on a playlist. Choosing 'play' a song is randomly selected from the playlist and played. Two songs are played, one after the other.

a What is the probability that the same song is played twice? (1 mark)

b How many songs would need to be on the playlist so that the probability of the same song being repeated is less than 5%? (1 mark)

6 *Taseefa has these ten cards.*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|

a What is the probability that the sum of the numbers on two cards, chosen randomly one after the other, is 7 if the cards are chosen:

i with replacement? (1 mark)

ii without replacement? (1 mark)

b What is the probability that a randomly chosen card is both odd and prime? (1 mark)

c Taseefa makes up a game where two cards are chosen one after the other with replacement. She wins if both cards are not both odd and prime. If Taseefa continued to play the game and bet money on the result would she make a profit or lose money? Justify your answer with appropriate calculations. (2 marks)

d If, in Taseefa's game, the first card is not replaced will it make a difference to whether she would make a profit or loss? Justify your answer. (2 marks)

**Your Feedback**

$\frac{\boxed{\phantom{00}}}{30} \times 100\% = \boxed{\phantom{00}}\%$

PAGES 189–190

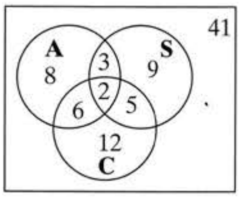
PAGE 198

Number of times 1 was spun
$= 24 - (8 + 6)$
$= 10$ ✓ (2 marks)

**4** a There are 6 blue marbles.
$P(\text{blue}) = \frac{2}{9} = \frac{6}{27}$
So there are 27 marbles altogether. ✓
Number of red marbles
$= 27 - (8 + 6)$
$= 13$
$P(\text{red}) = \frac{13}{27}$ ✓ (2 marks)

b $P(\text{blue or red}) = \frac{1}{4} + \frac{2}{5}$
$= \frac{13}{20}$ ✓
$P(\text{green})$
$= 1 - P(\text{blue or red})$
$= 1 - \frac{13}{20}$
$= \frac{7}{20}$ ✓ (2 marks)

**5**




a Total Year 9 students = 86
Number representing in swimming
$= 3 + 9 + 2 + 5$
$= 19$
$P(\text{swimming}) = \frac{19}{86}$ ✓ (1 mark)

b $P(\text{all 3 sports}) = \frac{2}{86}$
$= \frac{1}{43}$ ✓ (1 mark)

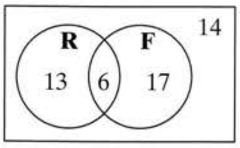
c $P(\text{only cross-country})$
$= \frac{12}{86}$
$= \frac{6}{43}$ ✓ (1 mark)

d $P(\text{athletics, cross-country, no swimming})$
$= \frac{6}{86}$
$= \frac{3}{43}$ ✓ (1 mark)

e Number representing in exactly 2 sports
$= 3 + 5 + 6$
$= 14$
$P(\text{exactly 2 sports})$
$= \frac{14}{86}$
$= \frac{7}{43}$ ✓ (1 mark)

f Number in swimming or athletics but not both
$= 9 + 5 + 6 + 8$
$= 28$
$P(\text{swimming or athletics but not both})$
$= \frac{28}{86}$
$= \frac{14}{43}$ ✓ (1 mark)

**6** 14 had not seen either movie.
Number who had seen a movie
$= 50 - 14$
$= 36$
Now $19 + 23 = 42$
$42 - 36 = 6$
So 6 students must have seen both. ✓




$P(\text{seen both}) = \frac{6}{50}$
$= \frac{3}{25}$ ✓ (2 marks)

(Total: 30 marks)

TWO-STEP CHANCE EXPERIMENTS

**SKILLS CHECK** PAGE 106

**1** a head, tail
b 6, 7, 8, 9

**2** a 1, 2, 3, 4, 5, 6
b A, E, G, N, R, S, T

**3** $P(\text{not k}) = 1 - \frac{1}{26}$
$= \frac{25}{26}$

**4**

| + | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

a $P(8) = \frac{5}{36}$

b $P(\text{less than 4}) = \frac{3}{36}$
$= \frac{1}{12}$

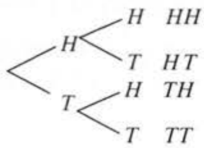
c $P(\text{greater than 6}) = \frac{21}{36}$
$= \frac{7}{12}$

d $P(\text{even}) = \frac{18}{36}$
$= \frac{1}{2}$

**5** a If the first pen is replaced there are 10 pens to choose from, 7 of which are blue.
$P(\text{blue}) = \frac{7}{10}$

b If the first pen is not replaced, 9 pens remain, 6 of which are blue.
$P(\text{blue}) = \frac{6}{9}$
$= \frac{2}{3}$

**6**




a $P(HH) = \frac{1}{4}$

b $P(TT) = \frac{1}{4}$

c $P(\text{one head and one tail})$
$= \frac{2}{4} = \frac{1}{2}$

d $P(\text{at least one head}) = \frac{3}{4}$

**7** a Total tees = 5 + 4
= 9
$P(\text{white}) = \frac{4}{9}$

b 8 tees remain, 3 of those are white.
$P(\text{white}) = \frac{3}{8}$

c If the tee is replaced 4 of the 9 are white.
$P(\text{white}) = \frac{4}{9}$

**8** a

| First marble | Second marble | Sample space |
|---|---|---|
| R | R | RR |
| | B | RB |
| | G | RG |
| B | R | BR |
| | B | BB |
| | G | BG |
| G | R | GR |
| | B | GB |
| | G | GG |

b $P(\text{one red, one blue}) = \frac{2}{9}$

**9** a

| First marble | Second marble | Sample space |
|---|---|---|
| R | B | RB |
| | G | RG |
| B | R | BR |
| | G | BG |
| G | R | GR |
| | B | GB |

b $P(\text{one red, one blue})$
$= \frac{2}{6} = \frac{1}{3}$

TWO-STEP CHANCE EXPERIMENTS

**INTERMEDIATE TEST** PAGE 107

**1** The probability cannot be $\frac{5}{2}$.
[The probability of any event must be between 0 and 1. $\frac{5}{2}$ is greater than one.]
$\therefore$ [C] ✓ (1 mark)

**2** If the first peg is red, 4 blue, 2 red and 2 green pegs remain.
Total remaining pegs
$= 4 + 2 + 2$
$= 8$
$P(\text{red peg}) = \frac{2}{8} = \frac{1}{4}$
$\therefore$ [B] ✓ (1 mark)

**3** Possible outcomes:
*HH*, *HT*, *TH*, *TT*
$P(TT) = \frac{1}{4}$
$\therefore$ [A] ✓ (1 mark)

**4**

| First ribbon | Second ribbon | Sample space |
|---|---|---|
| P | P | PP |
| | P | PP |
| | B | PB |
| P | P | PP |
| | P | PP |
| | B | PB |
| B | P | BP |
| | P | BP |
| | B | BB |

$P(BB) = \frac{1}{9}$
$\therefore$ [B] ✓ (1 mark)

**5** a

| First calf | Second calf | Sample space |
|---|---|---|
| B | B | BB |
| | R | BR |
| | W | BW |
| B | B | BB |
| | R | BR |
| | W | BW |
| R | B | RB |
| | B | RB |
| | W | RW |
| W | B | WB |
| | B | WB |
| | R | WR |

✓✓ (2 marks)

b $P(BB) = \frac{2}{12}$
$= \frac{1}{6}$ ✓ (1 mark)

c $P(\text{at least 1 is red}) = \frac{6}{12}$
$= \frac{1}{2}$ ✓
(1 mark)

d $P(\text{black}) = \frac{1}{3}$ ✓ (1 mark)
[There are three calves to choose from, 1 of these is black.]

**6**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 2 | 3 | 4 | 5 |
| 2 | 1 | 0 | 1 | 2 | 3 | 4 |
| 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| 4 | 3 | 2 | 1 | 0 | 1 | 2 |
| 5 | 4 | 3 | 2 | 1 | 0 | 1 |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 |

$P(1 \text{ or } 3) = \frac{16}{36} = \frac{4}{9}$ ✓

$P(2, 4 \text{ or } 5) = \frac{14}{36} = \frac{7}{18}$ ✓

Paris has a greater chance of winning because the probability of getting a 1 or a 3 is higher than the probability of getting a 2, 4 or 5. ✓ (3 marks)

**7** a

| First pen | Second pen | Sample space |
|---|---|---|
| Bk | B | BkB |
| | R | BkR |
| B | Bk | BBk |
| | R | BR |
| R | Bk | RBk |
| | B | RB |

✓

$P(\text{Black and Blue}) = \frac{2}{6}$
$= \frac{1}{3}$ ✓

[BkB or BBk] (2 marks)

b $P(BkB) = \frac{1}{2}$ ✓ (1 mark)
[The first pen was black so only consider those elements of the sample space that begin with black.]

**8** a $P(\text{B is first}) = \frac{1}{5}$ ✓ (1 mark)
[There are 5 cards that could be first and B is one of those.]

b $P(\text{C is second}) = \frac{1}{5}$ ✓
(1 mark)

[There are 5 cards that could be second and C is one of those.]

c

| | | | |
|---|---|---|---|
| AB | AC | AD | AE |
| BA | BC | BD | BE |
| CA | CB | CD | CE |
| DA | DB | DC | DE |
| EA | EB | EC | ED |

✓ (1 mark)

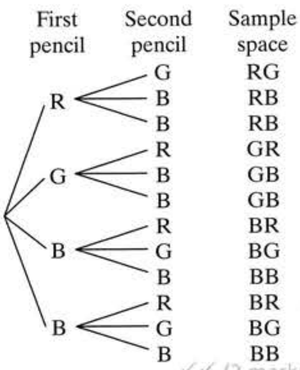
d $P(\text{DE or ED}) = \frac{2}{20}$
$= \frac{1}{10}$ ✓ (1 mark)

e If the first card was replaced there would be 5 more possible pairs.
[AA, BB, CC, DD, EE]
$P(\text{DE or ED}) = \frac{2}{25}$ ✓ (1 mark)
(Total: 20 marks)

TWO-STEP CHANCE EXPERIMENTS

**ADVANCED TEST** PAGE 108

**1** a




✓✓ (2 marks)

b i $P(BB) = \frac{2}{12}$

$= \frac{1}{6}$ ✓ (1 mark)

ii $P$(same colour)

$= P(BB)$

$= \frac{1}{6}$ ✓ (1 mark)

iii $P$(first is red) $= \frac{1}{4}$ ✓ (1 mark)

iv $P$(exactly 1 blue)

$= \frac{8}{12}$

$= \frac{2}{3}$ ✓ (1 mark)

v $P$(neither is blue)

$= P$(RG or GR)

$= \frac{2}{12}$

$= \frac{1}{6}$ ✓ (1 mark)

vi $P$(neither red)

$= \frac{6}{12}$

$= \frac{1}{2}$ ✓ (1 mark)

vii $P$(GB or BG)

$= \frac{4}{12}$

$= \frac{1}{3}$ ✓ (1 mark)

c If pencil is replaced:

$P(BB) = \frac{4}{16}$

$= \frac{1}{4}$ ✓

So, the probability of getting two blue pencils is greater if the first pencil is replaced. ✓ (2 marks)

**2** a $P$(Cath's mug is blue)

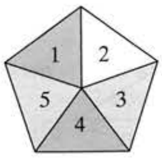
$= \frac{2}{6} = \frac{1}{3}$ ✓ (1 mark)

b $P$(Cath's mug now blue)

$= \frac{2}{5}$ ✓ (1 mark)

c If the colour of Jack's mug is not known, then Cath has an equal chance of getting any of the mugs so she has two chances in six of getting a blue one. But, if we know that Jack's mug is white, Cath can't get that mug so there are now only five mugs from which she can choose. She has two chances in five of getting a blue one. ✓ (1 mark)

**3**




a It doesn't matter what number is spun first, there is one chance in five of the same number occurring again.

$P$(same number) $= \frac{1}{5}$ ✓ (1 mark)

b It doesn't matter which number is spun first, there is one chance in five of spinning a number that when added to the first number will make 6.

$P$(adding to 6) $= \frac{1}{5}$ ✓ (1 mark)

c $P$(not the same number)

$= 1 - \frac{1}{5}$

$= \frac{4}{5}$

[Half will have the first number greater than the second.]

$P$(first number is greater)

$= \frac{2}{5}$ ✓ (1 mark)

**4** a $P(RR) = \frac{3}{10} \times \frac{2}{9}$

$= \frac{1}{15}$ ✓ (1 mark)

b $P$(neither is green)

$= \frac{5}{10} \times \frac{4}{9}$

$= \frac{2}{9}$ ✓

$P$(at least one green)

$= 1 - P$(neither is green)

$= 1 - \frac{2}{9}$

$= \frac{7}{9}$ ✓ (2 marks)

c $P$(exactly one is green)

$= P(G\tilde{G}) + P(\tilde{G}G)$

$= \frac{5}{10} \times \frac{5}{9} + \frac{5}{10} \times \frac{5}{9}$

$= \frac{5}{9}$ ✓ (1 mark)

**5** a $P$(same song) $= \frac{1}{10}$

[It doesn't matter which song is played first.] ✓ (1 mark)

b $5\% = \frac{1}{20}$

So if there were 20 songs the probability would be 5%. There would need to be 21 songs. ✓ (1 mark)

**6** a i If the card is replaced there are 10 possible cards that could be chosen first and 10 that could be chosen second.

Number of outcomes

$= 10 \times 10$

$= 100$

Outcomes that have a sum of 7:

$1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2$ and $6 + 1$

$P$(sum of 7)

$= \frac{6}{100}$

$= \frac{3}{50}$ ✓ (1 mark)

ii Without replacement there are only 9 cards that could be chosen second.

Number of outcomes
$= 10 \times 9$
$= 90$
$P(\text{sum of } 7) = \frac{6}{90}$
$= \frac{1}{15}$ ✓ (1 mark)

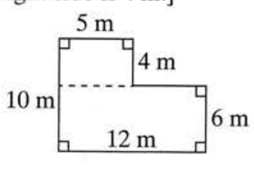
b Odd card numbers are 1, 3, 5, 7 and 9.
Prime card numbers are 2, 3, 5 and 7.
Numbers both odd and prime are 3, 5 and 7.
$P(\text{odd and prime}) = \frac{3}{10}$ ✓
(1 mark)

c $P(\text{not odd nor prime})$
$= 1 - \frac{3}{10}$
$= \frac{7}{10}$
$P(\text{both not odd nor prime})$
$= \frac{7}{10} \times \frac{7}{10}$
$= \frac{49}{100}$ ✓
Taseefa will lose money. The probability of winning is less than one-half. ✓
(2 marks)

d If the first card is not replaced:
$P(\text{both not odd nor prime})$
$= \frac{7}{10} \times \frac{6}{9}$
$= \frac{7}{15}$ ✓
No, it will not make a difference. The probability of winning is still less than half. ✓ (2 marks)
(Total: 30 marks)

## SAMPLE EXAM PAPER 1
*Part A: Multiple Choice* PAGE 110

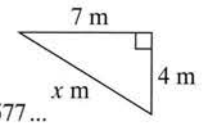
1 [The total length of the right-hand side must be the same as the left-hand side, 10 m. So the missing measurement on the right side is 4 m.]




$A = 5 \times 4 + 12 \times 6$
$= 20 + 72$
$= 92$
Area is 92 m$^2$.
$\therefore$ [A] ✓ (1 mark)

2 If $k = -3$,
$2k^2 = 2 \times (-3)^2$
$= 2 \times 9$
$= 18$
$\therefore$ [A] ✓ (1 mark)

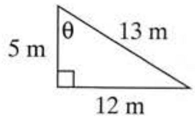
3 $x^2 = 7^2 + 4^2$
$= 65$
$x = \sqrt{65}$
$= 8.0622577\ldots$




Of the options the value of $x$ is closest to 8.
$\therefore$ [A] ✓ (1 mark)

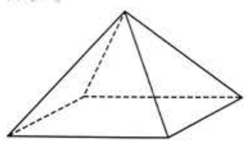
4 $0.000476 = 4.76 \times 10^{-4}$
[The decimal point moves 4 places.]
$\therefore$ [B] ✓ (1 mark)

5




$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
$= \frac{12}{13}$
$\therefore$ [D] ✓ (1 mark)

6 D is a pyramid not a prism.
$\therefore$ [D] ✓ (1 mark)



[A is a rectangular prism (or cube), B is an octagonal prism and C is a triangular prism.]

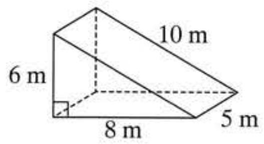
7 $835 = 840$ (2 significant figures)
$\therefore$ [D] ✓ (1 mark)
[The third figure is not required. Because it is 5 the second figure must be rounded up. The original number was close to 800 not 80 so the answer must be close to 800 not 80.]

8 [Melanie works 6 hours at 1½ times the normal rate of pay.]
Earnings $= 6 \times 1.5 \times \$18.60$
$= \$167.40$
$\therefore$ [C] ✓ (1 mark)

9 One microsecond is one-millionth of a second. $\frac{1}{1000000}$
$\therefore$ [D] ✓ (1 mark)

10 $7 - 2(x - 3) = 7 - 2x + 6$
$= 13 - 2x$
$\therefore$ [C] ✓ (1 mark)

11

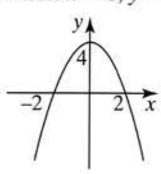



[The base of the prism is a triangle.]
Base: $A = \frac{1}{2} \times 8 \times 6$
$= 24$
area of base = 24 m$^2$
$V = Ah$
$= 24 \times 5$
$= 120$
volume = 120 m$^3$
$\therefore$ [A] ✓ (1 mark)

12 $A = 6s^2$
$= 6 \times 5^2$
$= 6 \times 25$
$= 150$
Surface area = 150 m$^2$
$\therefore$ [C] ✓ (1 mark)

13 $\frac{392}{21.3 \times 4.8} = \frac{392}{102.24}$
$= 3.8341158\ldots$
$= 3.8$ (1 d.p.)
$\therefore$ [A] ✓ (1 mark)

14 $y = 4 - x^2$ is a parabola.
It is 'upside down'.
When $x = 0$, $y = 4$




$\therefore$ [B] ✓ (1 mark)