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1. Simplify the following

$$(a) \quad \frac{6a^2bc}{3ab^2} = \frac{2ac}{b}$$

$$(b) \quad \frac{12ab}{a^2c} = \frac{12b}{ac}$$

$$(c) \quad \frac{10-2t}{5t} \text{ will not simplify}$$

$$(d) \quad \frac{a^2b-ab^2}{a-b} = \frac{ab(\cancel{a-b})}{\cancel{a-b}} = ab$$

$$(e) \quad \frac{3x^2y-12y}{x-2} = \frac{3y(x^2-4)}{x-2}$$

$$= \frac{3y(\cancel{x-2})(x+2)}{\cancel{x-2}}$$

$$= 3y(x+2)$$

$$(f) \quad \frac{4x^2yz-6xy^2z+2xyz^2}{2xy} = \frac{\cancel{2xy}z(2x-3y+z)}{\cancel{2xy}}$$

$$= z(2x-3y+z)$$

$$(g) \quad \frac{9-3x}{3x-9} = \frac{-3(x-3)}{3(x-3)}$$

$$= -1$$

$$(h) \quad \frac{x^2+2x-15}{3-x} = \frac{(x+5)\cancel{(x-3)}}{-(\cancel{x-3})}$$

$$= -(x+5)$$

$$(i) \quad \frac{x^2+2x-15}{2x^2+7x-15} = \frac{(x-3)\cancel{(x+5)}}{(2x-3)\cancel{(x+5)}}$$

$$= \frac{x-3}{2x-3}$$

$$(j) \quad \frac{4t^2-36}{t^2-3t} = \frac{4(\cancel{t-3})(t+3)}{t(\cancel{t-3})}$$

$$= \frac{4(t+3)}{t}$$

$$(k) \quad \frac{x^2+10x+25}{x^2-25} = \frac{(x+5)^2}{(x-5)\cancel{(x+5)}}$$

$$= \frac{x+5}{x-5}$$

$$(l) \quad \frac{6+7x-3x^2}{3x^2-x-2} = \frac{-(\cancel{3x+2})(x-3)}{(\cancel{3x+2})(x-1)}$$

$$= \frac{-(x-3)}{x-1}$$

$$= \frac{3-x}{x-1}$$

2. Perform the addition or subtraction. Simplify as much as possible.

$$(a) \quad \frac{4}{m} + \frac{2}{n} = \frac{4n+2m}{mn}$$

$$(b) \quad \frac{n}{m} + \frac{2}{3m} = \frac{3n+2}{3m}$$

$$(c) \quad \frac{2}{x} - \frac{1}{4x} - \frac{3}{5x} = \frac{40-5-12}{20x} = \frac{27}{20x}$$

$$(d) \quad \frac{1}{x^2} + \frac{2}{4x} + \frac{1}{5} = \frac{10+5x+2x^2}{10x^2}$$

$$(e) \quad \frac{3}{x+2} + \frac{x}{x-2} = \frac{3(x-2)+x(x+2)}{(x+2)(x-2)}$$

$$= \frac{x^2+5x-6}{(x+2)(x-2)}$$

$$= \frac{(x+6)(x-1)}{(x+2)(x-2)}$$

$$(f) \quad \frac{x-2}{4} - \frac{x+1}{3} = \frac{3(x-2)-4(x+1)}{12}$$

$$= \frac{-x-10}{12}$$

$$(g) \quad \frac{2(x+2)}{5} + \frac{3(4-x)}{4} = \frac{8(x+2)+15(4-x)}{20} \quad (h)$$

$$= \frac{-7x+76}{20}$$

$$y^2 - \frac{y+2}{y} = \frac{y^3 - (y+2)}{y}$$

$$= \frac{y^3 - y - 2}{y}$$

$$(i) \quad \frac{2-x}{x^2-16} + \frac{3}{x+4} = \frac{(2-x)+3(x-4)}{(x-4)(x+4)} \quad (j)$$

$$= \frac{2x-10}{(x-4)(x+4)}$$

$$\frac{x-3}{x^2-4x-5} - \frac{2(2x+1)}{2x^2-11x+5}$$

$$= \frac{x-3}{(x-5)(x+1)} - \frac{4x+2}{(2x-1)(x-5)}$$

$$= \frac{(x-3)(2x-1) - (x+1)(4x+2)}{(x-5)(x+1)(2x-1)}$$

$$= \frac{-2x^2 - 13x + 1}{(x-5)(x+1)(2x-1)}$$

$$(k) \quad \frac{1}{x^3-9x} + \frac{4}{x-3} - \frac{5}{x+3}$$

$$= \frac{1+4x(x+3)-5x(x-3)}{x(x-3)(x+3)}$$

$$= \frac{-x^2+27x+1}{x(x-3)(x+3)}$$

$$(l) \quad \frac{x^2-5x}{2x^2+7x+3} + \frac{4x^2-1}{x^2-2x-15}$$

$$= \frac{x(x-5)}{(2x+1)(x+3)} + \frac{4x^2-1}{(x-5)(x+3)}$$

$$= \frac{x(x-5)(x-5) + (4x^2-1)(2x+1)}{(2x+1)(x-5)(x+3)}$$

$$= \frac{(x^3-10x^2+25x) + (8x^3+4x^2-2x-1)}{(2x+1)(x-5)(x+3)}$$

$$= \frac{9x^3-6x^2+23x-1}{(2x+1)(x-5)(x+3)}$$

3. Simplify the following

$$(a) \quad \frac{7a^2b}{5c} \times \frac{2ac}{b^2} = \frac{7a^3bc}{5b^2c}$$

$$= \frac{7a^3}{5b}$$

$$(b) \quad \frac{4a^2b}{3c} \div \frac{2a}{3b} = \frac{4a^2b}{3c} \times \frac{3b}{2a}$$

$$= \frac{12a^2b^2}{6ac}$$

$$= \frac{2ab^2}{c}$$

$$(c) \quad \frac{3x+9}{5} \times \frac{10}{x+3} = \frac{30(x+3)}{5(x+3)}$$

$$= 6$$

$$(d) \quad \frac{16x^2y^2z^4}{5x} \div \frac{x^2}{2} = \frac{16x^2y^2z^4}{5x} \times \frac{2}{x^2}$$

$$= \frac{32x^2y^2z^4}{5x^3}$$

$$= \frac{32y^2z^4}{5x}$$

$$\begin{aligned}
 \text{(e)} \quad & \frac{x^2 - 4x}{x^2 - 4} \times \frac{x^2 + 4x + 4}{x^2 - 2x - 8} \\
 &= \frac{x \cancel{(x-4)} (x+2)^2}{(x-2) \cancel{(x+2)} \cancel{(x-4)} (x+2)} \\
 &= \frac{x}{x-2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & \frac{x^2 + 3x + 2}{x^2 - 7x + 12} \div \frac{2x^2 + 8x + 8}{x^2 + 7x + 6} \\
 &= \frac{x^2 + 3x + 2}{x^2 - 7x + 12} \times \frac{x^2 + 7x + 6}{2x^2 + 8x + 8} \\
 &= \frac{(x+1)(x+2)}{(x-3)(x-4)} \times \frac{(x+6)(x+1)}{2(x+2)^2} \\
 &= \frac{(x+6)(x+1)^2}{2(x+2)(x-3)(x-4)}
 \end{aligned}$$

4. Simplify the following

$$\begin{aligned}
 \text{(a)} \quad & \frac{2 - \frac{1}{x}}{\frac{2}{x} - 5} \\
 &= \frac{\frac{2x-1}{x}}{\frac{2-5x}{x}} \\
 &= \frac{2x-1}{\cancel{x}} \times \frac{\cancel{x}}{2-5x} \\
 &= \frac{2x-1}{2-5x}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{\frac{3}{x-2} + 1}{5 - \frac{1}{x+2}} \\
 &= \frac{3 + (x-2)}{5(x+2) - 1} \\
 &= \frac{x+1}{5x+9} \times \frac{x+2}{x+2} \\
 &= \frac{(x+1)(x+2)}{(x-2)(5x+9)}
 \end{aligned}$$

## Binomial expansions

1. Expand using Pascal's Triangle

$$\begin{aligned}
 \text{(a)} \quad & \text{Expand } (x+3)^4 \\
 & (x+3)^4 = 1x^4 + 4x^3(3) + 6x^2(3)^2 + 4x(3)^3 + 1(3)^4 \\
 & = x^4 + 12x^3 + 54x^2 + 108x + 81
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \text{Expand } (2x-3)^6 \\
 & (2x-3)^6 = 1(2x)^6 + 6(2x)^5(-3) + 15(2x)^4(-3)^2 + 20(2x)^3(-3)^3 + 15(2x)^2(-3)^4 + 6(2x)(-3)^5 + 1(-3)^6 \\
 & = 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4869x^2 - 2916x + 729
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & \text{Expand } \left(x + \frac{1}{x^2}\right)^4 \\
 & \left(x + \frac{1}{x^2}\right)^4 = 1x^4 + 4x^3\left(\frac{1}{x^2}\right) + 6x^2\left(\frac{1}{x^2}\right)^2 + 4x\left(\frac{1}{x^2}\right)^3 + 1\left(\frac{1}{x^2}\right)^4 \\
 & = x^4 + 4x + \frac{6}{x^2} + \frac{4}{x^5} + \frac{1}{x^8}
 \end{aligned}$$

$$\text{(d)} \quad \text{Expand } (2a - b^2)^6$$

$$(2a - b^2)^6 = 1(2a)^6 + 6(2a)^5(-b^2) + 15(2a)^4(-b^2)^2 + 20(2a)^3(-b^2)^3 + 15(2a)^2(-b^2)^4 + 6(2a)(-b^2)^5 + 1(-b^2)^6$$

$$= 64a^6 - 192a^5b^2 + 240a^4b^4 - 160a^3b^6 + 60a^2b^8 - 12ab^{10} + b^{12}$$

(e) Find the 4<sup>th</sup> term in  $(x - 4)^8$

$$T_{r+1} = {}^nC_r (a)^{n-r} (b)^r$$

$$T_4 = {}^8C_3 (x)^{8-3} (-4)^3 = \frac{8!}{3!5!} x^5 (-64) = -\frac{8 \times 7 \times 6}{6} \times 64x^5 = -3584x^5$$

(f) Find the 5<sup>th</sup> term in  $\left(\frac{2x}{3} + \frac{3}{x}\right)^{11}$

$$T_{r+1} = {}^nC_r (a)^{n-r} (b)^r$$

$$T_5 = {}^{11}C_4 \left(\frac{2x}{3}\right)^{11-4} \left(\frac{3}{x}\right)^4$$

$$= \frac{11!}{4!7!} \left(\frac{2x}{3}\right)^7 \left(\frac{3}{x}\right)^4$$

$$= 330 \times \frac{128x^7}{3^7} \times \frac{3^4}{x^4} = \frac{14080}{9} x^3$$

(g) Determine the Coefficient of  $x^5$  in  $(2x - 1)^{10}$

The  $x^5$  term is the 6<sup>th</sup> term ( $n - r = 5$ , as  $n = 10$ ,  $r = 5$ , it is the 6<sup>th</sup> term)

$$T_{r+1} = {}^nC_r (a)^{n-r} (b)^r$$

$$T_6 = {}^{10}C_5 (2x)^{10-5} (-1)^5$$

$$= -\frac{10!}{5!5!} \times 32x^5$$

$$= -252 \times 32x^5$$

$$= -8064x^5$$

(h) Determine the Coefficient of  $x^3$  in  $(7 - 3x)^7$

The  $x^3$  term is the 4<sup>th</sup> term (as  $r = 3$ , it is the 4<sup>th</sup> term)

$$T_{r+1} = {}^nC_r (a)^{n-r} (b)^r$$

$$T_4 = {}^7C_3 (7)^{7-3} (-3x)^3$$

$$= \frac{7!}{3!4!} (2401) (-27x^3)$$

$$= -35 \times 2401 \times 27x^3 = -2268945x^3$$

(i) Determine the Coefficient of  $x^2$  in  $\left(x^2 + \frac{1}{x}\right)^{10}$

The  $x^2$  term is the 7<sup>th</sup> term ( $2(n - r) - r = 2$ , as  $n = 10$ ,  $20 - 2r - r = 2$ , then  $r = 6$ )

$$T_{r+1} = {}^nC_r (a)^{n-r} (b)^r$$

$$T_7 = {}^{10}C_6 (x^2)^{10-6} \left(\frac{1}{x}\right)^6$$

$$= \frac{10!}{6!4!} x^8 x^{-6}$$

$$= 210x^2$$

