## KEYWORDS

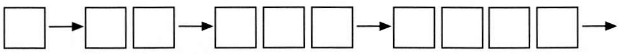
| | |
|---|---|
| **Algebra** | **Patterns** |
| **Expand** | **Power** |
| **Evaluate** | **Pronumerals** |
| **Expression** | **Relationship** |
| **Formula** | **Simplify** |
| **Indices** | **Substitution** |

# Number Patterns and Pronumerals

The study of number patterns is very important in the understanding of pronumerals.

### For Example

1 By considering the pattern below, made of matchsticks, complete the table.



| Number of squares | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Number of matchsticks | | | | | | | | | |

2 In your own words, write a rule that relates the number of matchsticks needed to build the pattern to the number of squares.

3 If $N$ stands for the number of matchsticks used to create $S$ squares in the pattern, write the rule in Question 2 using pronumerals.

4 Using the rule, find the number of matchsticks needed to create 30 squares.

5 Using the rule, find the number of squares created if 240 matchsticks are used.

1

| Number of squares | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Number of matchsticks | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |

2 Number of matchsticks
= 4 × number of squares in the pattern

3 $N = 4 \times S$
where $N$ = number of matchsticks
and $S$ = number of squares.

4 Using the rule found in Question 3, substitute $S = 30$ and find $N$.

$$N = 4 \times S$$
$$= 4 \times 30$$
$$= 120$$

Therefore, 120 matchsticks are needed to build a pattern with 30 squares.

5 Using the rule found in Question 3, substitute $N = 240$ and find $S$.

$$N = 4 \times S$$
$$240 = 4 \times S$$
$$S = 60$$

Therefore, 60 squares can be built with 240 matchsticks.

Note:

- $N$ and $S$ are called **pronumerals**.
- Pronumerals stand for numerals. In the above example, the pronumeral $N$ stands for the number of matchsticks and $S$ stands for the number of squares.
- In algebra, a rule is called a **formula**. For the above example, the formula is $N = 4 \times S$, where $N$ stands for the number of matchsticks used in the pattern and $S$ stands for the number of squares.

## For Example

1 Find a formula relating $x$ and $y$ in the following pattern:

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 3 | 5 | 7 | 9 | 11 |

1 It is observed that 'to obtain the number in the second row you need to multiply the number in the first row by 2 and then add 1', as follows:

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 3 | 5 | 7 | 9 | 11 |
| | $3 = 1 \times 2 + 1$ | $5 = 2 \times 2 + 1$ | $7 = 3 \times 2 + 1$ | $9 = 4 \times 2 + 1$ | $11 = 5 \times 2 + 1$ |

Therefore, the rule is $y = 2 \times x + 1$.

## For Example

1 Complete the table using the formula $N = 3 \times t - 1$:

| $t$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $N$ | | | | | |

1 When $t = 1$, $N = 3 \times 1 - 1 = 2$
$t = 2$, $N = 3 \times 2 - 1 = 5$
$t = 3$, $N = 3 \times 3 - 1 = 8$
$t = 4$, $N = 3 \times 4 - 1 = 11$
$t = 5$, $N = 3 \times 5 - 1 = 14$

Therefore, the completed table is as follows:

| $t$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $N$ | 2 | 5 | 8 | 11 | 14 |

**Definition**

A pronumeral represents a number and may take any numerical value. For example, $x = 2$ and $y = 5$.

# Operations on Pronumerals

The four operations of arithmetic (+, −, × and ÷) have the same meaning in algebra as they have in arithmetic:

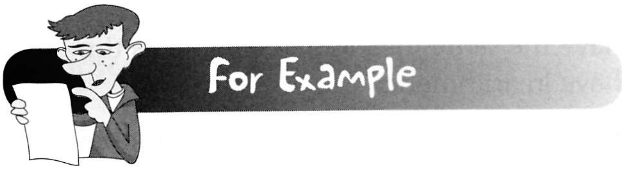
- $a + b$ means the **sum** of the numbers represented by the pronumerals $a$ and $b$. The actual value of $a + b$ can be found only if the values of $a$ and $b$ are known. If $a = 7$ and $b = 3$ then $a + b = 7 + 3 = 10$.
- $a - b$ means the **difference** of the numbers represented by the pronumerals $a$ and $b$. The value of $a - b$ can be found only if the values of $a$ and $b$ are known. If $a = 10$ and $b = 6$ then $a - b = 10 - 6 = 4$.
- $ab$ means the **product** (i.e. $ab = a \times b$) of the numbers represented by the pronumerals $a$ and $b$. The value of $ab$ can be found only if the values of $a$ and $b$ are known. If $a = 2$ and $b = 6$ then $ab = 2 \times 6 = 12$.
- $\frac{a}{b}$ means the quotient (i.e. $\frac{a}{b} = a \div b$) of the numbers represented by the pronumerals $a$ and $b$. The value of $\frac{a}{b}$ can be found only if the values of $a$ and $b$ are known. If $a = 12$ and $b = 4$ then $\frac{a}{b} = a \div b = \frac{12}{4} = 12 \div 4 = 3$.

Note: $ab$ is the shorthand way of writing $a \times b$ (that is, $a \times b = ab$). $\frac{a}{b}$ means $a \div b$ (that is, $\frac{a}{b} = a \div b$). $ab$ is called the simplified form. $a \times b$ is called the expanded form.

## Multiplication of Pronumerals

When multiplying pronumerals, the multiplication sign, ×, is often omitted.

For example, $a \times b$ is written as $ab$
$4 \times a = 4a$
$a \times b \times c = abc$




Simplify the following algebraic expressions:

| | | | |
|---|---|---|---|
| **1** | $3 \times b$ | **2** | $a \times 5$ |
| **3** | $2 \times a \times b$ | **4** | $5a \times 2b$ |
| **5** | $a \times a$ | **6** | $b \times 3a \times 4$ |
| **7** | $3 \times a \times 2a$ | | |

**1** $3 \times b = 3b$ [Omit the multiplication sign. Note the number is always written first.]

**2** $a \times 5 = 5a$ [Always write the number first.]

**3** $2 \times a \times b = 2ab$ [This expression could be written as $2ba$ because $ab = ba$.]

**4** $5a \times 2b = 10ab$ [$5 \times 2 = 10$, $a \times b = ab$. Multiply the numbers first and then write the letters.]

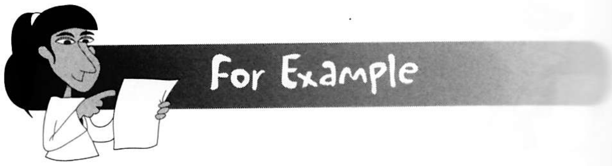
**5** $a \times a = a^2$ [$a \times a$ is not written as $aa$ but as $a^2$.]

**6** $b \times 3a \times 4 = 12ba$

[$b \times a = ba$, $3 \times 4 = 12$ Remember this expression could be written as $12ab$ since $ba = ab$.]

**7** $3 \times a \times 2a = 6a^2$

[Remember: multiply numbers first; $a \times a = a^2$ and not $aa$.]




Write the following expressions in expanded form:

| | | | | | |
|---|---|---|---|---|---|
| **1** | $4b$ | **2** | $3abc$ | **3** | $3b^2$ |
| **4** | $ab^2c$ | **5** | $(2a)^2$ | | |

**1** $4b = 4 \times b$

**2** $3abc = 3 \times a \times b \times c$

**3** $3b^2 = 3 \times b \times b$

**4** $ab^2c = a \times b \times b \times c$

**5** $(2a)^2 = 2a \times 2a = 2 \times a \times 2 \times a$

## *Substitution in Algebraic Expressions*

In substitution we replace the pronumeral with its numerical value (which must be given) and find the value of an arithmetic expression.


If $a = 2$, $b = 4$ and $c = 5$ evaluate:

| | | | |
|---|---|---|---|
| **1** | $ab$ | **2** | $a + b$ |
| **3** | $\frac{b}{2}$ | **4** | $c - a$ |
| **5** | $2b - c$ | **6** | $abc$ |
| **7** | $3a^2$ | **8** | $\frac{2b + 4}{a}$ |
| **9** | $c(b - a)$ | | |

**1** $ab = a \times b$
$= 2 \times 4$
$= 8$

[$ab = a \times b$.]
[Remember that $a = 2$ and $b = 4$.]

**2** $a + b = 2 + 4$
$= 6$

3 $\frac{b}{2} = \frac{4}{2}$ [Note $\frac{b}{2} = b \div 2$]
$= 4 \div 2$
$= 2$

4 $c - a = 5 - 2$
$= 3$

5 $2b - c = 2 \times b - c$ [Note order of operations. Do multiplication first.]
$= 2 \times 4 - 5$
$= 8 - 5$
$= 3$

6 $abc = a \times b \times c$
$= 2 \times 4 \times 5$
$= 40$

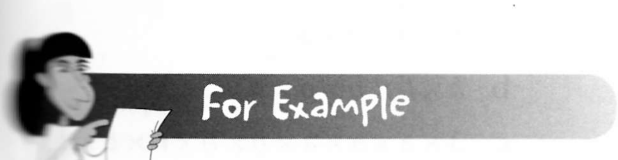
7 $3a^2 = 3 \times a \times a$ [$3a^2 \neq (3a)^2$ but $3a^2 = 3 \times a \times a$]
$= 3 \times 2 \times 2$
$= 12$

8 $\frac{2b + 4}{a} = \frac{2 \times b + 4}{a}$
$= \frac{2 \times 4 + 4}{2}$
$= \frac{8 + 4}{2}$
$= \frac{12}{2}$ [$\frac{12}{2}$ means $12 \div 2$]
$= 6$

9 $c(b - a) = 5 \times (4 - 2)$
$= 5 \times 2$
$= 10$

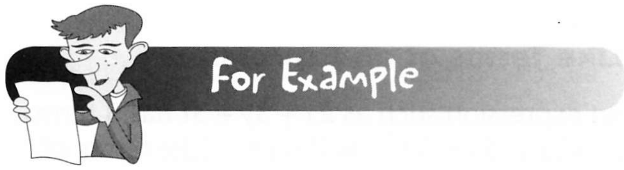
## Substitution into Simple Formulae

A formula combines at least two pronumerals. The value of one of the pronumerals can be found by substituting numbers for the other pronumerals.



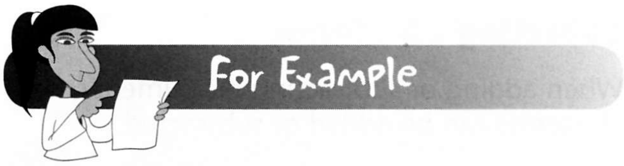

$T = 4n - 1$, find the value of $T$ when $n = 5$.

$T = 4n - 1$
$= 4 \times n - 1$
$= 4 \times 5 - 1$ [Replace $n$ by 5.]
$= 20 - 1$
$= 19$



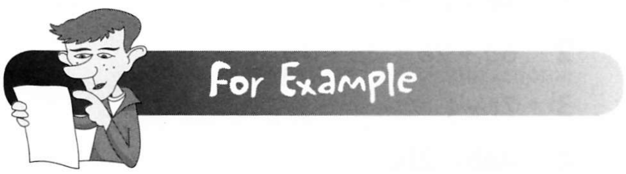

1 If $P = 2(n + b)$, and $n = 28$, $b = 12$, find $P$.

1 $P = 2(n + b)$
$= 2(28 + 12)$
$= 2(40)$
$= 2 \times 40$ [$2(40) = 2 \times 40$]
$= 80$




1 If $K = 4a^2$, find the value of $K$ when $a = 3$.

1 $K = 4a^2$
$= 4 \times a \times a$
$= 4 \times 3 \times 3$
$= 36$




1 If $x = ut + \frac{1}{2}at^2$, and $u = 12$, $t = 3$ and $a = 8$, find $x$.

1 $x = ut + \frac{1}{2}at^2$
$= 12 \times 3 + \frac{1}{2} \times 8 \times 3^2$
$= 12 \times 3 + \frac{1}{2} \times 8 \times 9$
$= 36 + 36$
$= 72$

## Like Terms

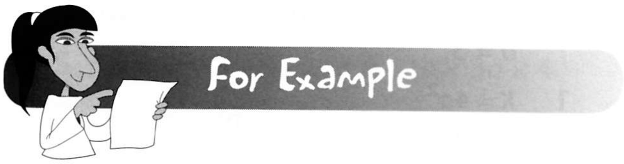
**Like terms** have the same pronumeral or pronumeral parts. For example, $x$, $-5x$, $4x$ and $20x$ are like terms.

### Like Terms of an Expression

An expression such as $2x + 3y + 3t$ has 3 terms. Similarly, $5x + 3x^2$ has 2 terms. Like terms of an expression are those that have the same pronumeral parts. For example, the expression $5x + 3y + 2x + 4x$ has 4 terms. $5x$, $2x$ and $4x$ are like terms of the expression. The following are examples of groups of like terms: $\{3y, y, -2y\}$, $\{ab, ba, 3ab\}$, $\{2x^2, -7x^2, 3x^2\}$. Similarly, $5x$, $3y$, $-2t$ are unlike terms since their pronumeral parts are all different.

### Collecting Like Terms

When adding or subtracting pronumerals, only like terms can be added or subtracted.




Simplify the following:

1 $3a + 2a + a$

2 $6a + 2b + 2a + 7b$

3 $7t - t$

4 $4ab - 2ba$

5 $5x^2 + 3x - 2x^2 + x$

6 $4t + k - 3t + 3k$

1 $3a + 2a + a = 6a$ [$a = 1a$; $(3 + 2 + 1)a$]

2 $6a + 2b + 2a + 7b = 8a + 9b$

[This expression could be rearranged so that like terms are grouped together.]

$6a + 2b + 2a + 7b = (6a + 2a) + (2b + 7b)$
$= 8a + 9b$

3 $7t - t = 7t - 1t$ [$t = 1t$]
$= 6t$

4 $4ab - 2ba = 2ab$ [$ab = ba$, therefore $4ab$ and $2ba$ are like terms.]

5 $5x^2 + 3x - 2x^2 + x = (5x^2 - 2x^2) + (3x + x)$
$= 3x^2 + 4x$

[Grouping like terms together; $x^2$ and $x$ are unlike terms.]

6 $4t + k - 3t + 3k = (4t - 3t) + (k + 3k)$

[Grouping like terms]

$= t + 4k$

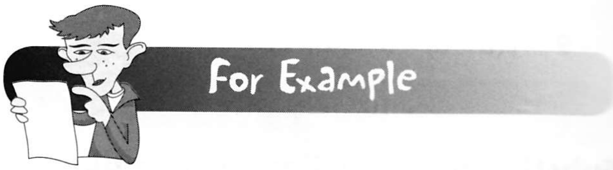
## Indices (Extension)

### Index Notation

$a \times a$ is written as $a^2$ ($a$ is squared)
$a \times a \times a$ is written as $a^3$ ($a$ is cubed)
$a \times a \times a \times a$ is written as $a^4$
(it is $a$ to the power of 4)

Note $a^m = \underbrace{a \times a \times a \times \ldots \times a}_{m \text{ times}}$ (it is called the $m$th power of $a$)

In the expression $a^m$ the $m$ is called the **index** or **power**. The plural of index is **indices**.




1 Evaluate:

a $2^3$ b $12^2$ c $3^4$

2 Write the following expressions in index form:

a $5 \times 5 \times 5 \times 5$

b $b \times b \times b \times a \times a$

c $3 \times a \times a \times a \times b \times b \times b \times b \times b$

1 a $2^3 = 2 \times 2 \times 2 = 8$

b $12^2 = 12 \times 12 = 144$

c $3^4 = 3 \times 3 \times 3 \times 3 = 81$

2 a $5 \times 5 \times 5 \times 5 = 5^4$

b $b \times b \times b \times a \times a = b^3a^2$ or $a^2b^3$

c $3 \times a \times a \times a \times b \times b \times b \times b \times b = 3a^3b^5$

### Laws of Indices

When multiplying, add indices. When dividing, subtract indices:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $a^0 = 1$

#### For Example

Simplify:

| | | | |
|---|---|---|---|
| 1 | $b^5 \times b^2$ | 2 | $a^8 \div a^2$ |
| 3 | $(x^4)^3$ | 4 | $(ab)^5$ |
| 5 | $(2x)^3$ | 6 | $y^0$ |
| 7 | $5a^2 \times 4a$ | 8 | $\dfrac{a^8 \times a^4}{a^3}$ |
| 9 | $12a^4b^5 \div 4a^2b^2$ | | |

1 $b^5 \times b^2 = b^{5+2} = b^7$

2 $a^8 \div a^2 = a^{8-2} = a^6$

3 $(x^4)^3 = x^{4 \times 3} = x^{12}$

4 $(ab)^5 = a^5b^5$

5 $(2x)^3 = 2^3 \times x^3 = 8 \times x^3 = 8x^3$

6 $y^0 = 1$

7 $5a^2 \times 4a = (5 \times 4)a^{2+1} = 20a^3$ [Multiply numbers first; $a = a^1$.]

8 $\dfrac{a^8 \times a^4}{a^3} = \dfrac{a^{12}}{a^3}$ [$\dfrac{a^{12}}{a^3}$ means $a^{12} \div a^3 = a^9$]

$= a^{12-3}$

$= a^9$

9 $12a^4b^5 \div 4a^2b^2 = 3a^{4-2}b^{5-2} = 3a^2b^3$ [Divide numbers first.]

## Division of Pronumerals

When dividing pronumerals, we can cancel components such as numbers and like terms.

#### For Example

Simplify:

| | | | |
|---|---|---|---|
| 1 | $18b \div 3$ | 2 | $12a \div 4a$ |
| 3 | $15xy^2t \div 5xy$ | 4 | $15m^2n \div 10mn^2$ |
| 5 | $25x^7y^4 \div 5x^3y^2$ | | |

1 $18b \div 3 = \dfrac{18b}{3}$ [Write in fractional form. Note $a \div b = \dfrac{a}{b}$]

$= \dfrac{\cancel{18}_6 \times b}{\cancel{3}_1}$ [Write it in expanded form.]

$= \dfrac{6b}{1}$ [Cancel common factors.]

$= 6b$ [$\dfrac{6b}{1}$ is written as $6b$.]

2 $12a \div 4a = \dfrac{12a}{4a}$ [Write in fractional form.]

$= \dfrac{\cancel{12}_3 \times \cancel{a}_1}{\cancel{4}_1 \times \cancel{a}_1}$ [Write in expanded form.]

$= \dfrac{3}{1}$ [Cancel common factor.]

$= 3$

**3** $15xy^2t \div 5xy = \dfrac{15xy^2t}{5xy}$

$= \dfrac{\cancel{15}_3 \times \cancel{x}_1 \times \cancel{y}_1 \times y \times t}{\cancel{5}_1 \times \cancel{x}_1 \times \cancel{y}_1}$

$= \dfrac{3yt}{1}$

$= 3yt$

**4** $15m^2n \div 10mn^2 = \dfrac{15m^2n}{10mn^2}$

$= \dfrac{\cancel{15}_3 \times \cancel{m}_1 \times m \times \cancel{n}_1}{\cancel{10}_2 \times \cancel{m}_1 \times \cancel{n}_1 \times n}$

$= \dfrac{3m}{2n}$

**5** $25x^7y^4 \div 5x^3y^2 = \dfrac{25}{5}x^{7-3}y^{4-2}$

[Use the rules of indices.]

$= 5x^4y^2$