Key definitions and concepts

Do you know all the key definitions and concepts for this chapter? Go through each term in the list and check that you know them all. Place a bookmark underneath each definition to cover up the one below and slide it down. This way you can focus on each definition by itself.

Aether was a hypothetical non-material fluid formerly hypothesised to permeate all space and having the property of propagating electromagnetic waves.

Aether wind was predicted as the result if the Earth moved through the aether. It was to attempt to detect this wind that the Michelson-Morley experiment was conducted. *See also* Michelson-Morley experiment.

Centripetal acceleration is directed towards the centre of a circle about which an object is moving.

Centripetal force is directed towards the centre of a circle required for an object to travel in a circular path. Gravity supplies the centripetal force to keep satellites in orbit.

Circular motion is the motion of an object in a circular path. If this is with constant speed it is called uniform circular motion (UCM).

Einstein, Albert German-born physicist best known for his work on relativity (1905 special relativity and 1915 general relativity). He also produced pioneering work on the photoelectric effect and Brownian motion. Was awarded the 1927 Nobel prize for his work on the photoelectric effect. *See also* photoelectric effect; special relativity.

Electromagnetic waves (radiation) are transverse waves composed of alternating electric and magnetic fields, the components of which are perpendicular to each other and to the direction of the energy flow.

Escape velocity is the velocity needed for an object to escape from the Earth (or other planet or moon). It depends on the radius and mass of the planet and the gravitational constant.

Frames of reference Objects or coordinate systems with respect to which we take measurements. *See also* inertial frames of reference; non-inertial frames of reference.

Geostationary orbits are those in which the satellite has a period of 24 hours and orbits in the equatorial plane about the Earth.

Geosynchronous orbits are those in which the satellite has a period of 24 hours. If the orbit is in the equatorial plane, the satellite appears to stay above the same point on the Earth and such an orbit is said to be geostationary.

g-forces are measured in units of the Earth's gravitational acceleration, *g*. For example, a force of 5*g* is equivalent to acceleration five times the acceleration due to gravity.

Gravitational acceleration is the acceleration due to gravity; equal to ~ 9.8 m.s^{-2} on Earth.

Gravitational constant The constant in Newton's Law of Universal gravitation (see also this entry). Equal to 6.67×10^{-11} N.m^2.kg^{-2}.

Gravitational field That region of space in which a mass experiences a force of attraction from other masses.

Gravitational potential energy is the work done to move an object from a very large distance away to a point in a gravitational field. For two masses m_1 and m_2 separated by a distance r, the gravitational potential energy is given by:

$$E_p = -G\frac{m_1 m_2}{r}$$

Gravity is the force of attraction between two or more masses.

Inertial frame of reference A frame of reference which is at rest or moving with constant velocity. It is a frame in which Newton's laws of motion are valid.

Length contraction is where the length of a moving rod appears to contract in the direction of motion relative to a stationary observer according to:

$$l_v = l_o\sqrt{1 - \frac{v^2}{c^2}}$$

Low-Earth orbits have an altitude that ranges from ~ 250 km to ~ 1000 km above the Earth's surface.

Mass dilation The mass of a moving object increases in relation to a stationary observer according to:

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Mass–energy Mass and energy are different forms of the same entity, i.e. $E = mc^2$

Measurement is the process of comparing some quantity such as length, mass or time to a selected standard and expressing the measured quantity as some factor of that standard.

Metre The distance travelled by light in a vacuum in the fraction $\frac{1}{299\,792\,458}$ of a second.

Michelson–Morley experiment Experiment conducted to measure the speed of the Earth through the aether. The inability of the experiment to find this speed led directly to Einstein's Theory of Special Relativity.

Non-inertial frame of reference An accelerated frame of reference. In such frames, inertial forces are present.

Orbit The path followed by an object travelling in space.

Orbital decay occurs when low altitude orbiting objects such as satellites and discarded 'space junk' re-enter the Earth's atmosphere and ultimately burn up.

Projectiles are any objects moving under the influence of gravity only.

Projectile motion is the motion of an object under the influence of a vertical force only (such as an object thrown through the air). Best analysed by dividing its motion into two components: horizontal component of motion with constant velocity; and vertical component of motion with constant acceleration.

Radio waves are long-wavelength (10^{-3} – 10^5 m) members of the electromagnetic spectrum.

Re-entry is the return of a spacecraft into the Earth's atmosphere and subsequent descent to Earth.

Relativity is a theory that describes matter, space and time and how they relate to each other. Einstein introduced the special theory of relativity in 1905 and the general theory of relativity in 1915 (the latter describes gravity). The principle of relativity is that the laws of physics are the same for all inertial observers. This results from the view that the speed of light is constant and is independent of the speed of the source or observer.

Simultaneity Where two or more events that are simultaneous for one observer are not necessarily simultaneous for observers in different inertial frames of reference.

Slingshot effect A method by which spacecraft can be accelerated by use of the planets. Relies on conservation of angular momentum.

Special relativity is the theory of relativity restricted to inertial frames of reference. *See also* Length contraction; Mass dilation; Mass-energy; and Time dilation.

Speed of light in a vacuum is constant and is independent of the speed of the source or the observer.

Thought experiments From the German term *Gedankenexperiment* are experiments 'conducted' entirely in a person's brain. Widely used by Einstein in his special theory of relativity.

Time dilation occurs where time in a moving frame appears to be slower relative to a stationary observer according to:

$$t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time travel The use of time dilation to allow trips to distant planets. The astronauts will age less than their counterparts on Earth and when they return to Earth they will return to a future many years ahead of their own.

Trajectory The path of a projectile.

Twin paradox A famous paradox of special relativity. Twins are separated at birth. One twin is placed in a spacecraft and leaves Earth. To the twin on Earth, the twin in the spacecraft ages slower. By symmetry, to the twin in the spacecraft, the twin on Earth ages slower! The paradox is resolved when it is realised that the twin in the spacecraft is in a non-inertial frame of reference and so the consequences of special relativity do not apply.

Universal gravitation The law that two or more masses attract each other according to

$$F = G\frac{m_1 m_2}{d^2}$$

Weight is the force on an object due to it being in a gravitational field.

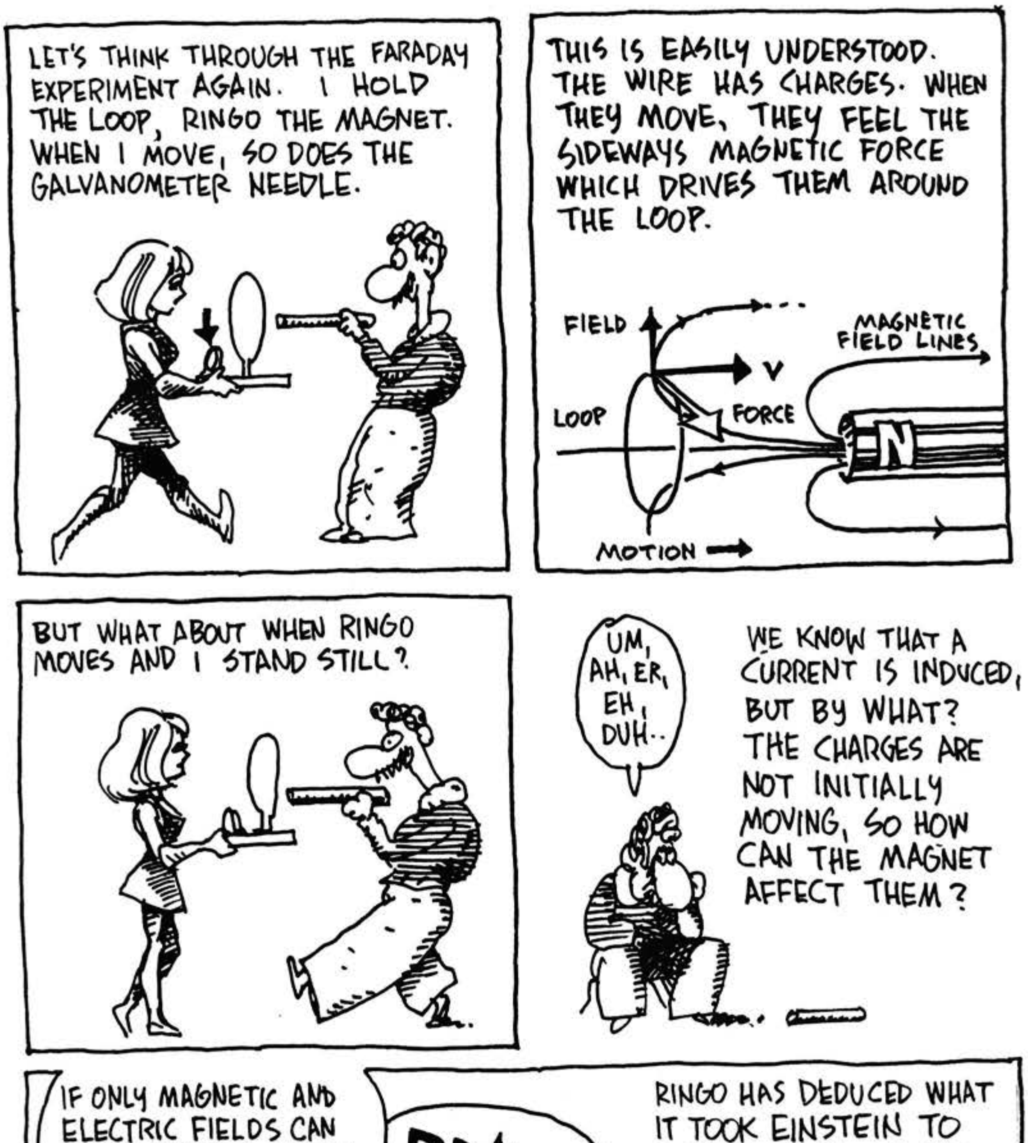
Chapter syllabus checklist

Are you able to answer every syllabus question for this chapter? Tick each question as you go through the list if you are able to answer it. If not, turn to the appropriate page in the guide as listed in the column to find the answer. Refer to page ix to check the meaning of the Board of Studies key words. You can also turn to the ***Excel*** syllabus summary notes at the back of the book for a summarised answer to each syllabus question.

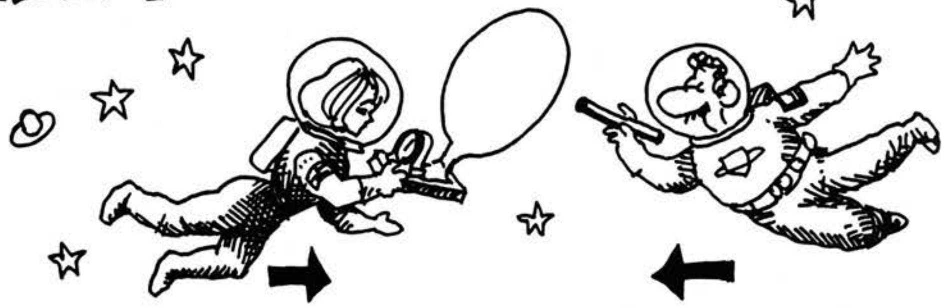
	For a complete understanding of this topic:	Page No.	✓
1	Can I define weight as the force on an object due to a gravitational field?	1	
2	Can I perform an investigation, and gather information to determine a value for acceleration due to gravity using pendulum motion, or computer assisted technology, and identify reasons for possible variations from the value 9.8 $m.s^{-2}$?	2	
3	Can I gather secondary information to predict the value of acceleration due to gravity on other planets?	3	
4	Can I analyse information using the expression $\vec{F} = m\vec{g}$ to determine the weight force for a body on Earth and for the same body on other planets?	4	
5	Can I explain that a change in gravitational potential energy is related to work done?	4	

•CHAPTER 20•

RELATIVITY

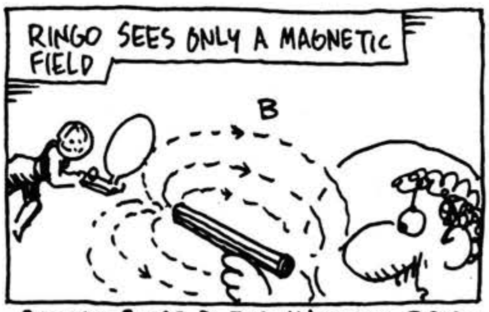

LET'S THINK THROUGH THE FARADAY EXPERIMENT AGAIN. I HOLD THE LOOP, RINGO THE MAGNET. WHEN I MOVE, SO DOES THE GALVANOMETER NEEDLE.
THIS IS EASILY UNDERSTOOD. THE WIRE HAS CHARGES. WHEN THEY MOVE, THEY FEEL THE SIDEWAYS MAGNETIC FORCE WHICH DRIVES THEM AROUND THE LOOP.
FIELD
V
MAGNETIC FIELD LINES
LOOP
FORCE
N
MOTION
BUT WHAT ABOUT WHEN RINGO MOVES AND I STAND STILL?
UM, AH, ER, EH, DUH..
WE KNOW THAT A CURRENT IS INDUCED, BUT BY WHAT? THE CHARGES ARE NOT INITIALLY MOVING, SO HOW CAN THE MAGNET AFFECT THEM?
IF ONLY MAGNETIC AND ELECTRIC FIELDS CAN MOVE CHARGES, THERE MUST HAVE BEEN AN ELECTRIC FIELD, TOO?
BRAVO!
RINGO HAS DEDUCED WHAT IT TOOK EINSTEIN TO REALIZE. EINSTEIN SAW THAT, DEPENDING ON WHO WAS MOVING, THE CURRENT IS SOMETIMES DUE TO A MAGNETIC FIELD AND SOMETIMES TO AN ELECTRIC FIELD.
CHANGING MAGNETIC FIELDS CAUSE ELECTRIC FIELDS!!

NOW AGAIN... ONE MORE TIME... WE'LL DO THE FARADAY EXPERIMENT... BUT THIS TIME IN **OUTER SPACE**, SO WE CAN'T TELL WHO IS "REALLY" MOVING. WE KNOW ONLY THAT WE ARE MOVING **RELATIVE** TO EACH OTHER.



I THINK I AM STATIONARY, AND RINGO IS MOVING. I DETECT A MAGNETIC FIELD, BUT IT CAN'T MOVE THE CHARGES, SO THERE MUST BE AN ELECTRIC FIELD ALSO, CAUSED BY THE CHANGING MAGNETIC FIELD.

RINGO THINKS HE IS STATIONARY AND I AM MOVING. HE DETECTS ONLY A MAGNETIC FIELD AND MOVING CHARGES, WHICH ACCOUNT FOR THE INDUCED CURRENT.



PHYSICISTS USE B FOR MAGNETIC FIELDS.

RINGO AND I DISAGREE ON WHAT FIELDS ARE PRESENT!

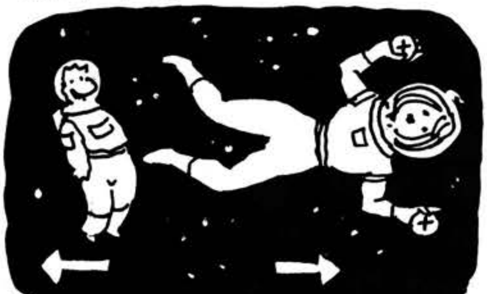
THIS IS THE HALLMARK OF RELATIVITY THEORY: TWO OBSERVERS LIKE RINGO AND ME, IF THEY ARE MOVING WITH RESPECT TO EACH OTHER, WILL DISAGREE ON THEIR MEASUREMENTS OF KEY PHYSICAL QUANTITIES OF THE UNIVERSE!

HERE'S AN EVEN SIMPLER ILLUSTRATION: A SINGLE CHARGE ZIPS THROUGH SPACE PAST RINGO:

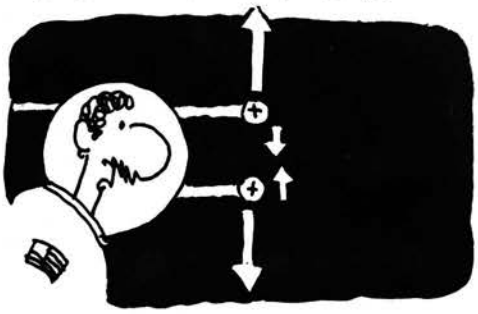
RINGO SEES A MOVING CHARGE — A CURRENT THAT GENERATES A MAGNETIC FIELD. THE NEEDLE OF RINGO'S COMPASS DEFLECTS!

BUT IF I AM MOVING WITH THE CHARGE, I SEE IT AS STATIONARY. THERE IS NO MAGNETIC FIELD, AND MY COMPASS IS NOT AFFECTED!

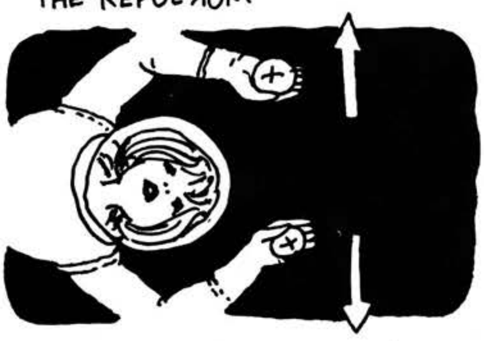
HERE'S THE FINAL DEMONSTRATION: WATCH CAREFULLY! I NOW CARRY **TWO** CHARGES SIDE BY SIDE PAST RINGO.



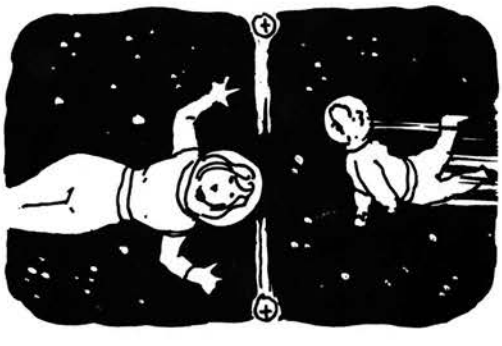
THEY REPEL EACH OTHER ELECTRICALLY — BUT RINGO SEES THEM MOVING: TWO PARALLEL CURRENTS, WHICH ATTRACT MAGNETICALLY!



BUT TO ME, THE CHARGES ARE STATIONARY, SO I SEE ONLY THE REPULSION.



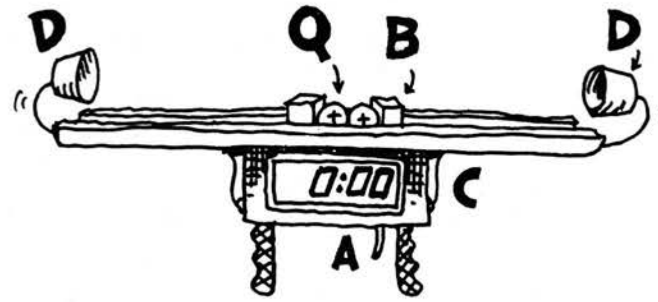
NOW I LET GO OF THE CHARGES. THEY FLY APART.



NOW THE STRANGE PART: RINGO SEES AN ATTRACTIVE MAGNETIC FORCE BETWEEN THE CHARGES, WHICH PARTLY OFFSETS THE REPULSIVE ELECTRIC FORCE — SO RINGO SEES THE CHARGES MOVE APART MORE SLOWLY THAN I DO!

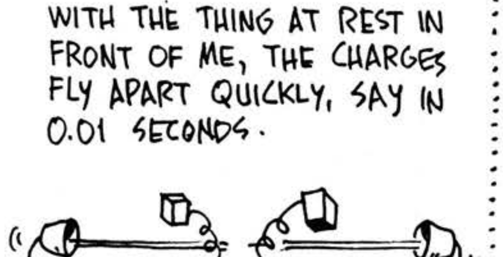
GOT THAT? RINGO, WHO IS MOVING WITH RESPECT TO ME, MEASURES THE CHARGES' OUTWARD VELOCITY TO BE **SLOWER** THAN I MEASURE IT!!

HERE IS AN APPARATUS FOR MEASURING HOW FAST THE CHARGES FLY APART.

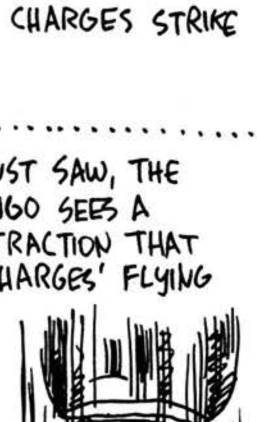


PULLING TRIGGER **A** RELEASES BLOCKS **B**, STARTING CLOCK **C** AND ALLOWING CHARGES **Q** TO FLY APART. CHARGES STRIKE CUPS **D**, STOPPING CLOCK **C**.

WITH THE THING AT REST IN FRONT OF ME, THE CHARGES FLY APART QUICKLY, SAY IN 0.01 SECONDS.



BUT, AS WE JUST SAW, THE SPEEDING RINGO SEES A MAGNETIC ATTRACTION THAT DELAYS THE CHARGES' FLYING APART.



RINGO MEASURES A **LONGER TIME** THAN I DO—SAY 0.02 SEC., FOR THE CHARGES TO FLY APART! HE ALSO NOTICES THAT MY CLOCK TICKS OFF ONLY 0.01 SEC. IN THE TIME IT TOOK HIS CLOCK TO REACH 0.02 SECONDS. CONCLUSION?

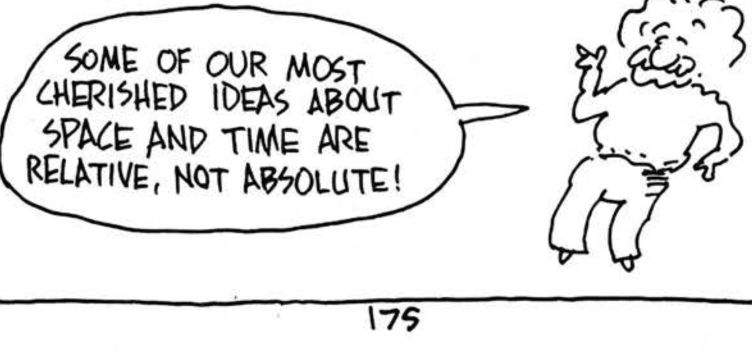
WHAT IS RINGO TO THINK? AS I SPEED BY, HE SEES MY CLOCK TICK OFF 0.01 SECONDS, WHILE HIS TICKS OFF TWICE AS MUCH. THERE IS ONLY ONE THING HE CAN CONCLUDE. RINGO DECIDES THAT—

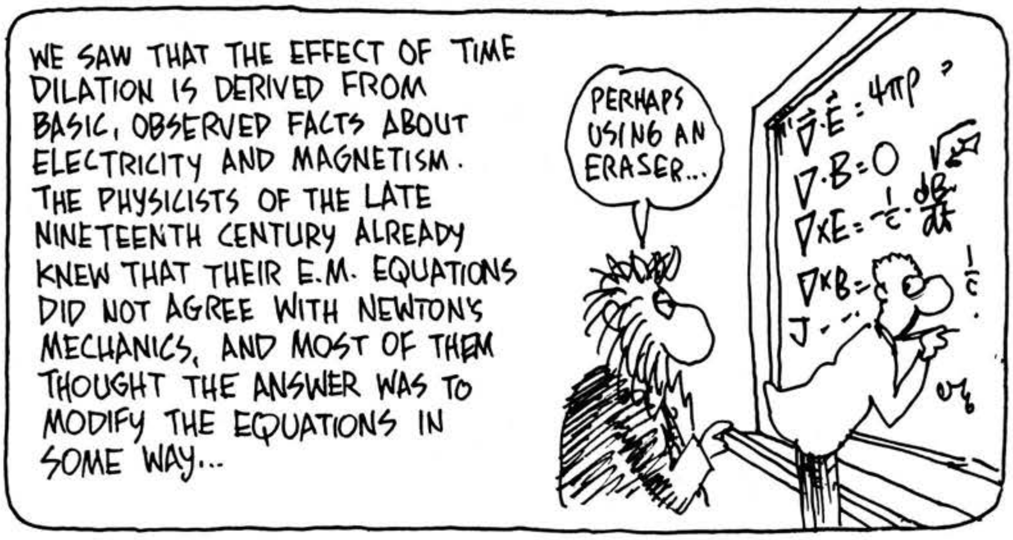
MY RAPID MOTION CAUSED MY TIME TO SLOW DOWN!!

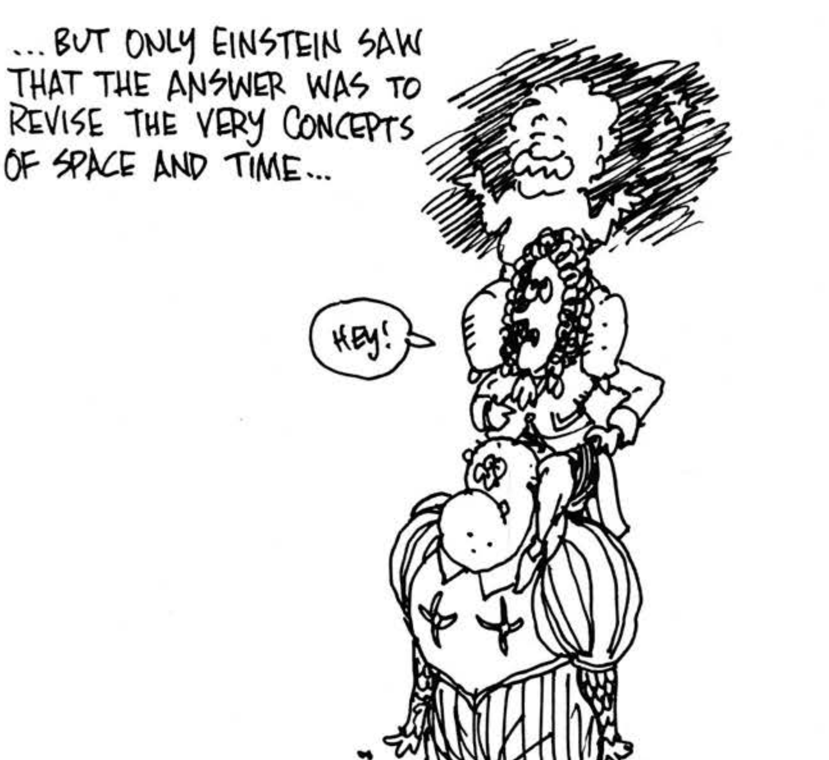
THAT IS JUST ONE OF THE WEIRD CONCLUSIONS OF RELATIVITY THEORY. AND THERE ARE MORE. ACCORDING TO EINSTEIN, A STATIONARY OBSERVER SEES THE FOLLOWING EFFECTS ON RAPIDLY MOVING OBJECTS:

* **TIME SLOWS DOWN**
* **LENGTHS DECREASE** (IN THE DIRECTION OF MOTION)
* **MASSES INCREASE**

IN OTHER WORDS—




WE SAW THAT THE EFFECT OF TIME DILATION IS DERIVED FROM BASIC, OBSERVED FACTS ABOUT ELECTRICITY AND MAGNETISM. THE PHYSICISTS OF THE LATE NINETEENTH CENTURY ALREADY KNEW THAT THEIR E.M. EQUATIONS DID NOT AGREE WITH NEWTON'S MECHANICS, AND MOST OF THEM THOUGHT THE ANSWER WAS TO MODIFY THE EQUATIONS IN SOME WAY...
PERHAPS USING AN ERASER...


...BUT ONLY EINSTEIN SAW THAT THE ANSWER WAS TO REVISE THE VERY CONCEPTS OF SPACE AND TIME...
HEY!

Describe the motion of one body relative to another.

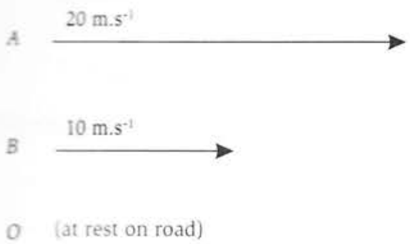


Figure 3.27 Relative velocity

3.2.1 Relative motion

Most of us have seen television pictures of astronauts 'space walking'. Although they are hurtling through space at thousands of kilometres per hour, they appear to be floating and, relative to their spacecraft, they appear to be at rest. Why is this?

As far back as Galileo in the 17th century, it was realised that *all motion, and hence all velocities, are relative*, that is, they are compared to a *frame of reference*, such as the spacecraft in the above example, the laboratory walls for your class experiments, a roadway, and so on. The velocity of an object thus depends upon what you measure it against. In the example of the astronauts, they are travelling at high speeds, but so too is their spacecraft, so relative to each other they are 'at rest'.

Similarly a driver and passenger in a car are at rest relative to the car, but are travelling at the car's speed relative to the roadway.

Relative velocity

Let's use the following example to illustrate the relativity of velocities. Consider two cars travelling at 20 $m.s^{-1}$ east and 10 $m.s^{-1}$ east *relative to the road*, as in Figure 3.27.

To a stationary pedestrian O on the footpath, the velocity of car A is 20 $m.s^{-1}$ east; this is written as $\vec{v}_{AO} = 20\ m.s^{-1}$ east. Similarly, the velocity of B relative to the observer O is given by $\vec{v}_{BO} = 10\ m.s^{-1}$ east.

How fast does car A appear to be travelling relative to the driver in car B? You could probably do this easily in your head and come up with $\vec{v}_{AB} = 10\ m.s^{-1}$ east. What you did, possibly without realising it, was to subtract the velocity of the observer B from the velocity of A.

Using a similar idea, you can show that the velocity of B relative to A is $-10\ m.s^{-1}$ east or $10\ m.s^{-1}$ west!

In general we can write: $\vec{v}_{AB} = \vec{v}_{AO} - \vec{v}_{BO} = \vec{v}_{AO} + \vec{v}_{OB} = \vec{v}_{A} - \vec{v}_{B}$

Following are further examples of the use of vector diagrams to solve problems on relative velocity. KCq page 152

Identify the usefulness of using vector diagrams to assist solving problems.

3.2.2 Using vector diagrams

Example 11

When travelling north at 15 $m.s^{-1}$ towards an intersection, a driver A notices another car approaching the intersection from the east (Figure 3.28). Given that the speed of the second car B is 20 $m.s^{-1}$ relative to the ground, calculate the velocity of B relative to A.

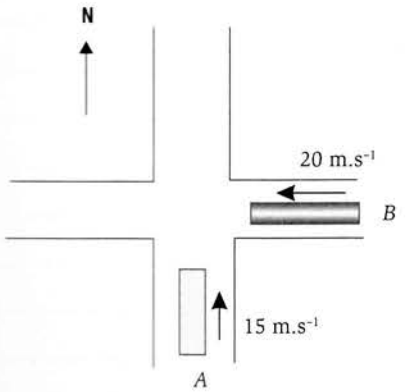


Figure 3.28 Two cars approaching an intersection

Solution

Our general equation becomes $\vec{v}_{BA} = \vec{v}_{B} - \vec{v}_{A}$. By direct measurement from the diagram (Figure 3.29) we have: $\vec{v}_{BA} = 25\ m.s^{-1}$ bearing 233 degrees.

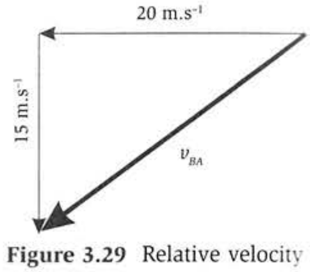


Figure 3.29 Relative velocity example

Example 12

A plane heads due north at 200 $km.h^{-1}$ while a wind blows from the west at 40 $km.h^{-1}$. Calculate the velocity of the plane relative to the ground.

Solution

The general equation gives: $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$ where P is the plane, A is the air and G is the ground.

We draw a scaled vector diagram (Figure 3.30) and let 1 cm = 20 km.h^{-1}

From Figure 3.30 we find:

$\vec{v}_{PG}$ = ~204 km.h^{-1}, bearing 11.3 degrees.

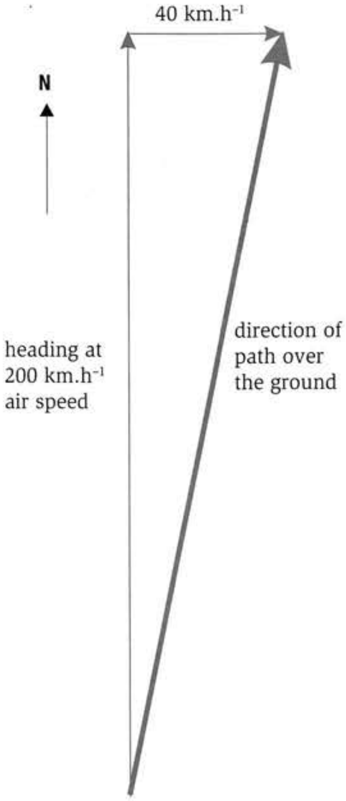


Figure 3.30 Velocity of plane relative to the ground

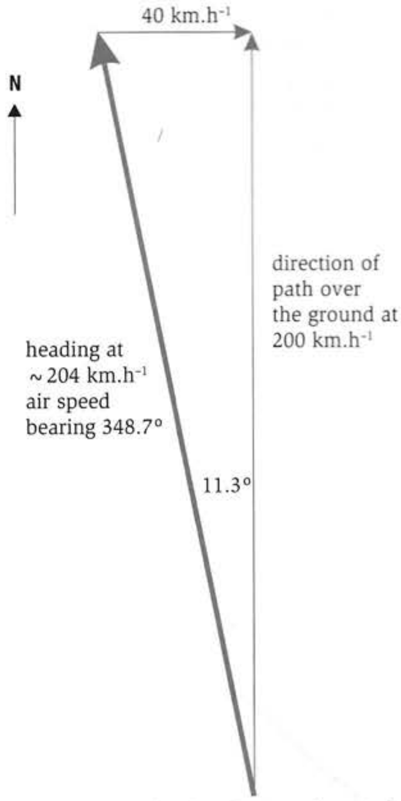


Figure 3.31 Pilot heads 'into the wind'

Example 13

In Example 13 it is evident that if the pilot simply headed due north, he/she would not arrive at the destination, as the plane would be 'blown off course'. Rather, the pilot would need to compensate for the wind. Calculate the direction and speed that the pilot would have to head, in order to arrive at the desired destination with a 'ground speed' of 200 km.h^{-1}.

Solution

The pilot would need to head 'into the wind' slightly, so that the *resultant* of the plane's motion and the motion of the wind, would be due north as shown in Figure 3.31.

The pilot would have to fly at 204 km.h^{-1}, bearing 348.7 degrees, to arrive at the required destination. KCq pages 152–153

EXTENSION

Components of vectors

As we saw earlier (p. 114), *a single resultant vector can replace any number of vectors*. The opposite of this is that a single vector can be *composed* from a number of others. These are called *components* of the original vector (Figure 3.32a).

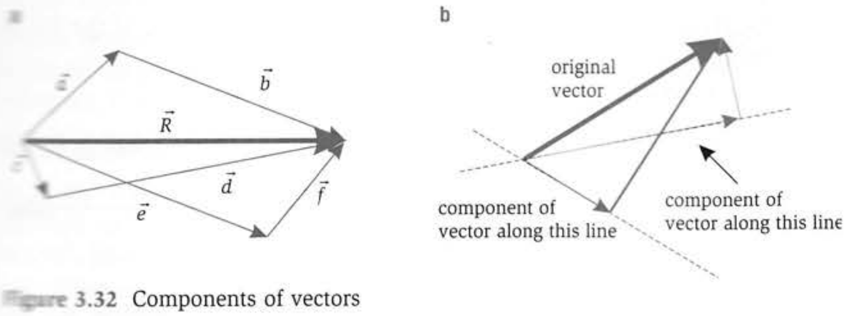


Figure 3.32 Components of vectors

The process of finding the components of vectors is called *resolving vectors*.

To find the component of a vector in a given direction, simply drop a line from the head of the vector to meet the line at right angles. The size of the component can then be measured from the scaled diagram (Figure 3.32b).

Rectangular components

Of prime importance are components that are perpendicular to each other. These are *rectangular components*.

In the special case of rectangular components, the size of the components can be found easily from trigonometry.

Consider Figure 3.33.

It follows from trigonometry that:

$\sin\theta = \frac{a_y}{a}$ and $\cos\theta = \frac{a_x}{a}$

That is:

$a_x = a\cos\theta$

$a_y = a\sin\theta$

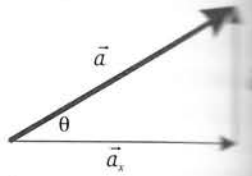


Figure 3.33 Rectangular components

From Pythagoras' Theorem we also have: $a = \sqrt{a_x^2 + a_y^2}$

Example 14

A plane flies at 250 km.h^{-1} bearing 45 degrees (that is, north-east). Calculate its northerly component and its easterly component of velocity.

Solution

Consider Figure 3.34. The easterly component is found from:

$v_x = v\cos 45 = 250\cos 45 = 177$ km.h^{-1}

Similarly, the northerly component is found from:

$v_y = v\sin 45 = 250\sin 45 = 177$ km.h^{-1}

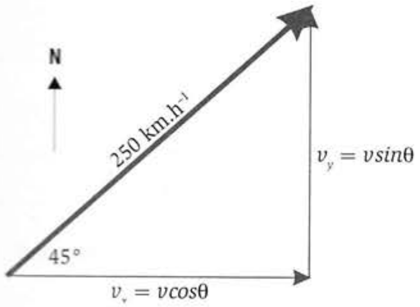


Figure 3.34 Velocity components

Example 15

A car is at rest on a hill as shown in Figure 3.35.

If the weight of the car is W, calculate the value of the component of the weight:

a acting at right angles to the hill and

b acting down the hill.

Solution

From Figure 3.35 we see that:

a The component of weight perpendicular to the plane of the hill is given by:

$F_{perp} = W\cos\theta$

b Similarly, the component of weight parallel to the plane of the hill is given by:

$F_{par} = W\sin\theta$

We will refer to these relationships later when we consider forces acting on cars going uphill or downhill.

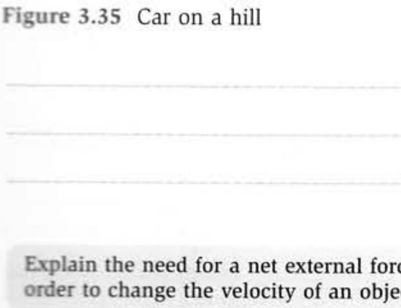


Figure 3.35 Car on a hill

Explain the need for a net external force to act in order to change the velocity of an object.

3.2.3 What is a force?

Now that we know something about the description of motion, we are in a position to investigate the causes of motion; that is, we look at forces—the study called *dynamics*. But what exactly is a force?

You should be aware from your previous study that:

a *force is a push or a pull.*

$$t_{ADA} = \frac{\frac{2l}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

It follows that $t_{ABA} \neq t_{ADA}$. If we replace the two swimmers with light rays and the motion of the raft through the water with the 'aether wind', we have the essence of the Michelson-Morley experiment. KCq page 48

Discuss the role of the Michelson-Morley experiment in making determinations about competing theories.

1.4.3 The role of experiments in science

Science progresses as a result of the validation of hypotheses by experimentation. From a hypothesis, predictions are made of what should happen if a particular experiment is performed. If, when the experiment is performed, the results are not in agreement with the prediction (and the same thing happens when the experiment is repeated), the hypothesis is incorrect. As we have seen, the Michelson-Morley experiment set out to measure the speed of the Earth through the aether. A null result showed the aether hypothesis to be invalid. Although Michelson and Morley did their experiment in 1887 and Einstein proposed his theory in 1905, it is likely Einstein did not know of the experiment, making no reference to it. The Michelson-Morley experiment, however, provided supporting evidence for Einstein's theory, allowing a choice between two conflicting theories, one requiring an aether and one that did not. To better understand the importance of the Michelson-Morley experiment, we need to look at the concept of *frames of reference.* KCq page 48

Outline the nature of inertial frames of reference.

1.4.4 Frames of reference

Frames of reference are objects or coordinate systems with respect to which we take measurements.

Position

In maths, the Cartesian coordinate system is used and position is referred to the axes x, y and z. In your experiments in class, the laboratory is your frame of reference.

Velocity

An object P travels with velocity v with respect to a reference frame S (Figure 1.26). Another frame, S', moves with velocity u relative to S. The velocity of P relative to S' is $\vec{v}\,' = \vec{v} - \vec{u}$. Velocity thus depends upon the reference frame.

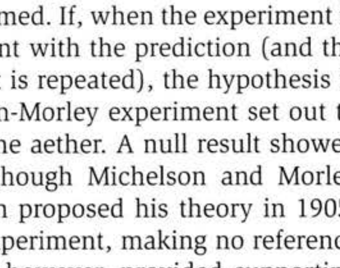


Figure 1.26: Frames of reference

Inertial frames of reference

An inertial frame of reference is one that is moving with constant velocity or is at rest (the two conditions being indistinguishable, see Newtonian relativity). In such reference frames, Newton's Law of Inertia holds. No frame is any more correct than any other, but some are simpler. The fixed stars are often taken as the best example of an inertial frame.

Non-inertial frames of reference

A non-inertial frame of reference is one that is accelerating. In such frames observers have to postulate the existence of 'forces' to maintain the validity of Newton's Laws. These are pseudo (fictitious) or inertial forces.[11] KCq page 48

11. Centrifugal force is an inertial force. A weightless observer in orbit around the Earth postulates an outward force to counteract the force of gravity.

FIRST-HAND INVESTIGATION Inertial and non-inertial frames of reference

Perform an investigation to help distinguish between non-inertial and inertial frames of reference.

AIM: To distinguish between inertial and non-inertial frames of reference.

THEORY: Inertial frames of reference are ones that are at rest or move with constant velocity. Non-inertial frames are those that are accelerating. Both types can be easily illustrated with a mass attached to a piece of string as you travel to school in a car, bus or train. (The mass and string arrangement is called a 'plumb bob'.)

METHOD: Hold one end of the string so that the mass hangs vertically. Observe what happens when the car (or bus or train):

- is stopped
- accelerates from rest
- travels at constant speed in a straight line
- slows down as it approaches a stop sign
- travels in a roundabout (i.e. travels in a curve).

RESULTS: The following typical results are observed:

- When the car is stopped, the string hangs vertically and the mass does not move.
- When the car accelerates from rest, the mass 'moves' in the opposite direction to the direction of movement of the car so that the string makes an angle with the vertical.
- Travelling at constant speed in a straight line, the string hangs vertically and the mass does not move.
- When the car slows down, the mass moves in the same direction as the direction of motion of the car so that the string makes an angle with the vertical.
- In a roundabout, the mass 'moves' away from the vertical in a direction opposite to the direction of the curved motion, i.e. if the car makes a right-had turn the mass moves to the left.

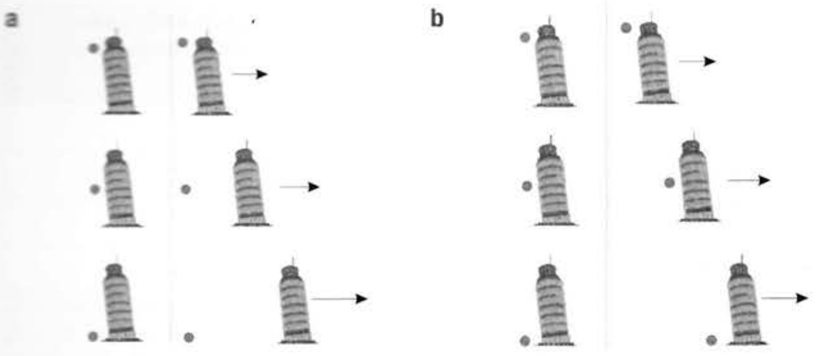
CONCLUSION: Inertial and non-inertial frames of reference can be illustrated from their effects on a mass hanging vertically from a string. The mass hangs freely in inertial frames and experiences force in non-inertial frames. KCq page 48

1.4.5 The principle of relativity

Discuss the principle of relativity.

Galilean relativity

Relativity did not begin with Einstein, but much earlier with Galileo and then Newton. The study of motion as we know it began with Galileo. This study necessarily involves the concepts of space and time. The view that motion must be relative, that is, it involves displacement of objects relative to some reference system, began with Galileo. Galileo's analysis of projectile motion led him to consider reference frames, which as we have seen, are what all measurements are compared to. For example, the desks and walls of the laboratory is your common frame of reference. Galileo was a strong advocate of the *heliocentric* model of the universe which has the Sun at the centre and all the planets revolving around it, contrast to the *geocentric* ('Earth centred') model current in his day. Galileo's opponents believed that if the Earth moved, then a stone dropped from a tower would be 'left behind' and fall away from



Stationary Earth Moving Earth

Galileo's critics argued that if the Earth moved, a stone dropped from the top of the tower would be left behind and land away from the tower's base.

Stationary Earth Moving Earth

Galileo argued that the stone and tower have the same horizontal velocity and so would fall near the base. An observe could not tell from the stone's motion whether the Earth moved or not.

Figure 1.27: Galileo and frames of reference

the tower's base as in Figure 1.27a. This did not happen (Figure 1.27b), and so Galileo's critics said this showed the Earth did not move about the Sun!

Galileo proposed that the stone did not fall behind because it shared the Earth's motion. The tower and the stone had the same horizontal velocity and because of the independence of the vertical and horizontal motions, the stone would fall close to the base (as actually occurred). Looking at the stone could not tell an observer whether the Earth moved or not.

In 1642 he even devised an experiment where an object was dropped from the 'crows nest' of a sailing ship. He showed that the object fell straight down relative to the mast, whether the ship was stationary or moving with constant velocity! (When the ship is moving, the object traces out a parabolic path relative to the background.)

As a result of these experiments and other *thought* experiments (defined later in this chapter), Galileo stated what become known as the principle of Galilean Relativity; that is, 'the laws of mechanics are the same for a body at rest and a body moving with constant velocity'.

We saw earlier that velocity is relative. What about acceleration? Suppose the object P in the previous Figure 1.26 is now undergoing a uniform acceleration. We have: $\vec{v}\,' = \vec{v} - \vec{u}$

Then:

$$\vec{a} = \frac{\Delta\vec{v}\,'}{\Delta t} = \frac{\Delta(\vec{v} - \vec{u})}{\Delta t} = \frac{\Delta\vec{v}\,'}{\Delta t} - \frac{\Delta\vec{u}\,'}{\Delta t} = \vec{a} - 0$$

since for constant velocity $\vec{u}$, the change is zero.

Newton took this idea further. He said that two observers travelling at relative velocity of $\vec{u}$ would see the same acceleration. Both observers will agree on the mass being the same[12] and so both will see the same form for the second law, that is: $\vec{F} = m\vec{a}$ and $\vec{F}\,' = m\vec{a}\,'$ so both frames are equivalent.

Newtonian relativity

The concept of a frame of reference took on more importance with the work of Newton. Since observers in inertial frames will get the same results for experiments based on Newton's Second Law, Newton extended this to become a more general statement. Newtonian relativity states that 'it is impossible to do any mechanical experiment, wholly within an inertial frame of reference that can tell you whether the frame is at rest or moving with constant velocity'.

Absolute motion

In his *Principia Mathematica* Newton defined absolute motion as 'the translation of a body from one absolute place to another', without expanding on what he meant by absolute place.

Similarly he defined absolute time as 'absolute true and mathematical time, of itself, and from its own nature flows equally without regard to anything external'. Time was regarded as independent of space. Newton was aware that motion was relative but believed that ultimately there was

12. This is true of speed much less than the speed of light (see later).

a fixed frame of reference to which all other motions could be compared. This frame was the aether. But as we have seen, all attempts to measure the motion of the Earth through the aether proved futile.

Attempts to explain the negative result of the Michelson-Morley experiment

To preserve the idea of the aether as an absolute frame of reference a number of proposals tried to account for the negative result (also called a null result) of the Michelson–Morley experiment.

- It was suggested that the Earth 'carried the aether along' with it so that there was no relative motion. Other observations (for example, the aberration of light), however, showed this to be incorrect.
- Hendrik Lorentz in Holland and George Fitzgerald in Ireland proposed that the length of the apparatus used by Michelson and Morley contracted in the direction of motion. (While this 'explained' the negative result, it was simply an *ad hoc* assumption with no physical basis.) Lorentz and Fitzgerald reached this conclusion as a result of their studies of Maxwell's equations of electromagnetism viewed from different frames of reference.

Poincaré's relativity

The investigations of Lorentz were restricted to electromagnetism and light, and it was a bold step to extend it to ordinary dynamics. But in 1904 Henri Poincaré introduced the 'Principle of Relativity', namely: The laws of physics are the same for a 'fixed' observer as for an observer who has a uniform motion of translation relative to him. This extended Newtonian-Galilean relativity to include the laws of electromagnetism (as well as the laws of mechanics).

In 1905 Einstein proposed a whole new theory of dynamics; the theory of special relativity (this theory is restricted to inertial frames of reference and neglects gravity[13]). KCq page 48

1.4.6 The special theory of relativity[14]

Describe the significance of Einstein's assumption of the constancy of the speed of light.

In 1905 Albert Einstein proposed that: The speed of light is constant and is independent of the speed of the source or the observer.

This premise explained the 'negative' result of the Michelson-Morley experiment and showed that the aether concept was not needed. As a consequence of this 'law of light' it can be shown that there is no such thing as an absolute frame of reference[15]. All inertial reference frames are equivalent. That is, all motion is relative.

Einstein's theory of special relativity represents one of the greatest changes in scientific thought since the time of Newton. It presents many apparently impossible conclusions; conclusions that appear to defy common sense. Einstein, however, described common sense as a 'deposit of prejudice laid down in the mind prior to the age of eighteen'. Because the effects of 'Einsteinian' relativity become obvious only at speeds approaching the speed of light, we are generally unaware of effects such as length contraction, mass dilation and time dilation. KCq page 49

13. Gravity is included in Einstein's General Theory of Relativity.
14. http://math.ucr.edu/home/baez/relativity.html. Relativity is explained on the web in popular science sites, animated graphics and tutorials. Another excellent site that give a nice 'simple' explanation of special relativity is at http//howstuffworks.com/relativity.htm.
15. Which had been the role of the aether, but it did not even perform that task, so there was no point having it!

Identify that if c is constant then space and time become relative.

1.4.7 Implications of the constancy of the speed of light: time and space are relative

Consider a spacecraft moving at a speed of $c/2$ (that is, half the speed of light) towards another planet. An astronaut in the spacecraft now flashes a light beam in the direction of the motion of the spacecraft. What is the speed of the light relative to the planet? Prior to Einstein we would have said $3c/2$ but now we know it is c. How can this happen?

To measure speed we need to measure distance and time. If c remains constant, then it follows that distance (length) and time must change! Space and time are relative concepts. (As we will see later, so too is mass.) To understand how these effects come about we have to review what we mean by measurement. KCq page 49

Discuss the concept that length standards are defined in terms of time in contrast to the original metre standard.

1.4.8 Measurement

Measurement is the process of comparing some quantity, such as length, mass or time to a selected standard and expressing the measured quantity as some factor of that standard. It follows then that all measured quantities are relative quantities.

The metre

The standard of length is the metre. This was originally defined as one ten millionth of the distance between the equator and the North Pole along the meridian passing through Paris. This 'distance' was marked on a platinum-iridium bar. Copies were made and sent throughout the world. All distances were compared to this standard.

Following advances in the accurate measurement of light wavelengths, this measure was changed to one defined the wavelength of the light emitted from the element krypton-86 when excited in a discharge tube. Since 1960, the metre has been defined in terms of time and velocity as 'the distance travelled by light in a vacuum in the fraction $\frac{1}{299\,792\,458}$ of a second'.

Strange as it may seem, our current standard of length is defined in terms of time. The emphasis on the processes of measurement became vital with relativity (and quantum mechanics). Our reality is what we measure it to be. Reality and observation cannot be separated. Remember this as we proceed.

Measuring length

It is a simple matter to measure the length, say, of a stationary object. But what if the object is moving? To measure the length of a moving object, such as a train passing a station, it is necessary to mark points on the station directly opposite the front and back of the train simultaneously. It is then a simple matter to measure the distance on the station between the points and so measure the length of the moving train. The measurement depends for its accuracy on all observers agreeing as to the simultaneity of marking the front and back of the train. But will they? (See 1.4.10). KCq page 49

Analyse and interpret some of Einstein's thought experiments involving mirrors and trains and discuss the relationship between thought and reality.

1.4.9 Relationship between thought and reality

Einstein used 'thought experiments' (from the German term *Gedankenexperiment*) to arrive at his ideas on special relativity. As the name suggests, thought experiments are not real experiments, but are 'performed' in the mind. The problem with them is that what we think will happen is largely determined by our previous experiences as the basis of what we often call 'common sense'—as we have seen Einstein equated which, as prejudice laid down in the mind prior to the age of eighteen. Nevertheless, because not all experiments could be tested because of technical limitations at the time, Einstein still used thought experiments.

One of his most famous was to imagine himself on a train travelling at the speed of light while holding a mirror at arms length in front of his face. He wondered if he would see his reflection in the mirror. There are two possibilities:

- He would see his reflection in the mirror.
- He would not see his reflection.

Both possibilities have difficulties.

If he were to see his image this would mean that light was travelling at c relative to the train (as expected on the aether model). This however would mean that an outside observer would see the light travelling at $2c$ relative to a stationary observer, violating Einstein's constancy of the speed of light! If he could not see his reflection this would mean that the light could not 'catch up' to the mirror so he could infer he was travelling at c without reference to an outside reference frame. This would violate the principle of relativity!

Such thought experiments assisted Einstein in his formulation of the special theory of relativity. KCq page 49

1.4.10 Simultaneity and the velocity of light

Explain qualitatively and quantitatively the consequence of special relativity in relation to the relativity of simultaneity.

Galileo tried to measure the velocity of light by using lanterns flashing between mountaintops. All he could conclude from his crude experiment was that the speed of light was extremely fast.

In 1676 the Dutchman Christiaan Huygens, using observations of the satellites of Jupiter made by the Danish astronomer Ole Roemer, calculated the speed of light to be 2×10^8 m.s^{-1}. Since then many experiments have yielded the value of $c = 3 \times 10^8$ m.s^{-1}. Although this speed is very fast, it is not infinite and hence must be taken into account when dealing with simultaneity.

Two events at A and B separated by a distance l will be simultaneous if the observer at A in Figure 1.28 records an event at A occurring at time t, and that from B occurring at time $t + \frac{l}{c}$. (Alternatively we can define two events A and B as simultaneous in a particular frame of reference, if light from these events arrives simultaneously at the mid-point between A and B.)

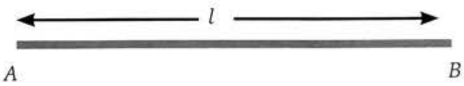


Figure 1.28: Simultaneous events

Another thought experiment

Assume two Einsteinian spaceships are travelling with relative velocity v parallel to each other. Further assume that observers O and O' are in the middle of their respective spaceships and that highly charged points A and A' and B and B' are directly opposite each other at some instant (Figure 1.29a).

Now assume that sparks jump between A and A' and B and B' so that the flashes are simultaneous for O. Will the events also be simultaneous for O'?

As shown in Figure 1.29b, in the finite time t the spacecraft would have moved relative to each other. O sees O' approaching the light from B' (still at c) and receding from the light from A', and hence O concludes that the events

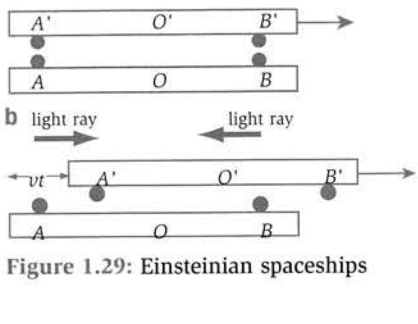


Figure 1.29: Einsteinian spaceships

are not simultaneous for O'. Conversely if the events were simultaneous for O' then O' would conclude that the events were not simultaneous for O. Observers in relative motion will disagree on the simultaneity of events separated in space. This is known as the relativity of simultaneity.

From our discussion on the measurements of moving objects and simultaneity, it should be clear that the lengths of moving objects depend upon the frame of reference from which they are measured! KCq page 49

Explain qualitatively and quantitatively the consequence of special relativity in relation to length contraction.

1.4.11 Length contraction

Length contraction is where the length of a 'moving' rod appears to contract in the direction of motion relative to a 'stationary' observer.

Lorentz-Fitzgerald contraction equation

$$l_v = l_o\sqrt{1 - \frac{v^2}{c^2}}$$

where l_v is the moving length, l_0 is the 'rest' length (that is, the length as measured by an observer at rest with respect to the rod) and v is the speed of the rod.

(This was Lorentz's original 'explanation' for the negative result of the Michelson–Morley experiment, but now it has a physical basis.)

Consider Figure 1.30, which shows how a 'flying saucer' would appear to different observers, one at rest with respect to the saucer and one who is moving relative to the saucer. Note that the contraction is in the direction of motion only (resulting in a flying 'ellipse'!).

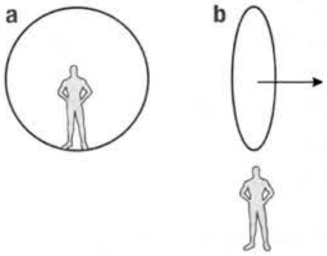


Figure 1.30: Flying saucers

The factor $\sqrt{1 - \frac{v^2}{c^2}}$ and its reciprocal $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ are central to special relativity.

Table 1.7 gives the values of these for a range of values of v (expressed as a fraction of the speed of light). We will refer back to this table in the examples to follow. KCq page 49

v	$\beta = \sqrt{1 - \frac{v^2}{c^2}}$	$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
0.1c	0.995	1.005
0.5c	0.886	1.155
0.75c	0.661	1.512
0.9c	0.436	2.29
0.99c	0.141	7.08
0.999c	0.0447	22.37

Table 1.7

Solve problems and analyse information using $l_v = l_o\sqrt{1 - \frac{v^2}{c^2}}$.

PROBLEM SOLVING On length contraction using $l_v = l_o\sqrt{1 - \frac{v^2}{c^2}}$

Example 24

A spacecraft moves away from the Earth. If an observer on the spacecraft measures some object to be 1.0 m long, calculate the length an observer on the Earth determines this object to be if:

a the spacecraft is moving at $0.1c$

b the spacecraft is moving at $0.999c$.

Solution We have the length contraction formula:

$$l_v = l_o\sqrt{1-\frac{v^2}{c^2}} = \beta l_o$$

a From Table 1.6: $l = 1.0 \times 0.995$ m $= 99.5$ cm

b From Table 1.6: $l = 1.0 \times 0.0447$ m $= 4.47$ cm!

We can see that according to the observer on Earth, the object on the spacecraft has shrunk (in length).[16] KCq page 49

1.4.12 Time dilation

Explain qualitatively and quantitatively the consequence of special relativity in relation to time dilation.

Time dilation is where the time in a 'moving' frame appears to go slower relative to a 'stationary' observer.

The time dilation equation is: $t_v = \dfrac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma t_0$

where t_v is the observed time for a 'stationary' observer and t_0 is the time for an observer travelling in the frame and

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

t_v is called the *proper time*. This is the time measured by an observer present at the same location as the events that indicate the start and end of an event. Time dilation today, is a well-accepted scientific fact (see 1.4.14 below). KCq page 49

EXTENSION

Proof of time dilation (another *gedankenexperiment*)

Consider the simple light 'clock' in Figure 1.31a. The clock operates by bouncing a light beam from mirrors. The clock registers one click for one complete up and down motion. When viewed by an observer travelling with the clock, the light follows the path in Figure 1.31a. From the point of view of an observer who sees the clock moving past at constant speed, the path is as in Figure 1.31b. This path is longer than that in Figure 1.31a and since both observers agree on the speed of light as being c then the outside observer must conclude that time lengthens!

Mathematically: In Figure 1.31a $t_0 = \dfrac{2l}{c}$ in Figure 1.30b Pythagoras' Theorem gives

$$(ct_{AB})^2 = (vt_{AB})^2 + l^2$$

$$t_{AB} = \frac{\frac{l}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_{AB} = \frac{\frac{2l}{c}}{\sqrt{1-\frac{v^2}{c^2}}} = t_v$$

$$\therefore t_v = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

'Real' clocks

Our simple 'light clock' seems to indicate that time dilates, but what about 'real clocks' that operate on quartz crystals or with gears and levers? How could they slow down? What about our own biological clocks for growth and aging? Special relativity states that all moving clocks experience time dilation, even our own biological clocks (see 1.4.15 Space travel). If this were not the case, then we could determine a discrepancy between 'light clocks' and 'mechanical clocks' or own 'body clocks' and so infer we were moving. This, however, violates the view that all inertial frames of reference are equivalent.

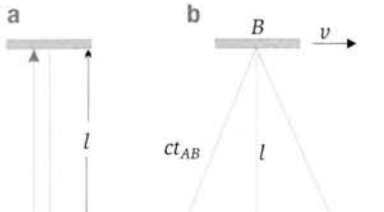


Figure 1.31: Time dilation

16. http://www.walter-fendt.de/ph11e/timedilation.htm. Physics applet that simulates time dilation.

Solve problems and analyse information using:

$$t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

PROBLEM SOLVING On time dilation using $t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma t_0$

Example 25

If one hour of time passes for the observer on the spacecraft, calculate how much time passes (on the spacecraft) from the point of view of the observer on Earth, if the velocity of the spacecraft is: **a** 0.5c. **b** 0.9c.

Solution We have the time dilation formula: $t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$

a We have $t_0 = 1\,\text{h}$. From Table 1.6 we then have $t_v = 1.0 \times 1.155$ or 1.155 hours (that is, 69 min 14 sec).

b From Table 1.6 we have: $t_v = 1.0 \times 2.29 = 2.29$ hours (that is, 137 min 24 sec).

The time, as measured by the observer on the Earth, has been lengthened (dilated). (For an observer in the spacecraft, his clock works perfectly correctly; he sees the clocks on the Earth going slow!) KCq page 49

Explain qualitatively and quantitatively the consequence of special relativity in relation to the equivalence between mass and energy.

1.4.13 Mass–energy

When we do work on an object we increase its kinetic energy ($E_k = \frac{1}{2}mv^2$). As the speed approaches c we still do work but the kinetic energy does not increase significantly. The work goes into increasing the object's mass according to Einstein's famous equation: $E = mc^2$

where $m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0$

Conservation of mass-energy

The Einstein equation shows that mass and energy are interchangeable—a Law of Conservation of Mass-Energy has now replaced the separate laws of mass and energy conservation. KCq page 49

Solve problems and analyse information using $E = mc^2$.

PROBLEM SOLVING On mass–energy using $E = mc^2$

Example 26

1 kg of water requires 4.18×10^5 J of energy to raise its temperature from 0°C to 100° C. Calculate the corresponding mass increase.

Solution $E = mc^2 \Rightarrow m = \frac{E}{c^2}$

$$= \frac{4.18 \times 10^5}{(3.0 \times 10^8)^2}$$

$$= 4.63 \times 10^{-12}\ \text{kg}$$

This example shows that the conversion of energy into mass (and mass into energy) can be ignored in physical changes. This is also true for chemical changes.

The conversion of mass into energy occurs in nuclear fission and fusion. In nuclear physics however, where particles can be accelerated to speeds close to the speed of light, the conversion of mass and energy cannot be ignored. Because the amounts of energy for individual reactions is small when expressed in SI units (joules), a new unit of energy is used—the electron volt.

The electron-volt

One electron-volt (eV) is the energy gained by an electron accelerated through a potential difference of one volt.

It can be shown that:

1 eV = 1.602×10^{-19} J 1 keV = 1.602×10^{-16} J

1 MeV = 1.602×10^{-13} J

Since mass and energy are equivalent, physicists often give the mass of subatomic particles in energy units/c^2. For example, the mass of a proton is stated as: $m_p = \dfrac{938}{c^2}$

Example 27

The mass of an electron and its antiparticle the positron are each 9.1×10^{-31} kg. In a collision between an electron and a positron the two particles are annihilated and two equal energy gamma rays are produced. Calculate the energy of the gamma rays.

Solution The energy of the gamma rays is found from:

$$
\begin{aligned}
E &= mc^2 \\
&= 2 \times 9.1 \times 10^{-31} \times (3.0 \times 10^{8})^2 \\
&= 1.638 \times 10^{-13}\text{ J or} \\
&= \frac{1.638 \times 10^{-13}}{1.6 \times 10^{-16}}\text{ keV} \\
&= 1024\text{ keV}
\end{aligned}
$$

Hence each gamma ray has energy of 512 keV. KCq page 49

1.4.14 Mass dilation

Explain qualitatively and quantitatively the consequence of special relativity in relation to mass dilation.

Not only do length and time depend on an object's speed but so too does its mass. The mass of a 'moving' object is greater than when it is 'stationary'. This effect is called mass dilation. As an object's speed increases, the mass of the object changes as shown in Figure 1.32 according to the mass dilation formula below.

Mass dilation equation is:

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0$$

where m_v is the mass for a 'moving' object and m_0 is the mass for that object when it is 'stationary'—the rest mass.

How do we know that mass must increase for a moving object relative to a stationary observer? Since c is the maximum speed in the universe then it follows that a steady force applied to an object cannot continue to accelerate the object indefinitely or else it would cause the object's speed to exceed c. This means the inertia, that is the resistance to acceleration of the object, must increase. But inertia is a measure of mass and so the mass increases causing the acceleration to get less and less so that the object never reaches c as indicated in Figure 1.33.

As with length and time, the mass measured by an observer travelling with the object is unchanged. KCq page 49

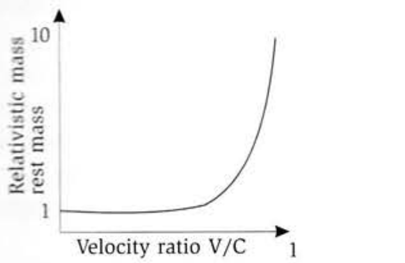


Figure 1.32: Effect of increasing speed on mass

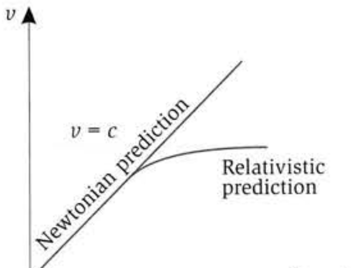


Figure 1.33: A comparison between the Newtonian and Einsteinian predictions of the effect of increasing the speed on the mass

Solve problems and analyse information using:

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

PROBLEM SOLVING On mass dilation using $m_v = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_o$

Example 28

A 1.0 kg mass is accelerated to a speed of 0.75c. Calculate the mass as determined by a stationary observer.

Solution $m_v = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_o$

and from Table 1.7 we find $\gamma = 1.512$ so relativistic mass $= 1.0 \times 1.512 = 1.512$ kg. KCq page 49

Analyse information to discuss the relationship between theory and the evidence supporting it, using Einstein's predictions based on relativity that were made many years before evidence was available to support them.

SECONDARY-SOURCE INVESTIGATION **Relationship between theory and evidence supporting it**

Many of Einstein's predictions were not able to be verified for years after he first postulated them. Mostly this was due to the lack of appropriate technology. Nevertheless scientists came to accept Einstein's work and in time all his predictions were experimentally corroborated.

How do we know that time dilates?

A number of experiments have conclusively proven time dilation actually happens. In one celebrated experiment, two extremely accurate atomic clocks were synchronised. One was then flown around the world in a plane and the other clock remained on the ground. When they were compared, the clock from the plane lagged slightly behind the one left behind. Although the difference was very small, it was measurable. A second famous experiment involves a subatomic particle called a muon that is created in the upper atmosphere by cosmic rays. Muons are unstable and disintegrate in a lifetime of 2.2 μs (in their reference frame). In this time, they should not be able to reach the Earth's surface regardless of their enormous speed of $\sim 0.999c$. Yet, they are found at the Earth's surface. Relative to observers on Earth, their lifetime is dilated ($= 22.37 \times 2.2 = 49.2$ μs), sufficient for them to reach the Earth.

NB: Relative to the muon, the atmosphere is contracted in the direction of motion to an extent that the muon has 'less' atmosphere to travel through before reaching the Earth's surface.

Mass dilation and mass-energy transformations

In linear accelerators and cyclotrons, designers need to account for the increasing mass of charged particles as they are accelerated to higher and higher speeds to ensure they are synchronised to continue to gain speed. The energy released in radioactive decay and nuclear reactors and explosions provides irrefutable evidence for the conversion of mass into energy. KCq page 50

Discuss the implications of mass increase, time dilation and length contraction for space travel.

1.4.15 Space travel and relativity

Science fiction often has time travel as a theme, where people can be transported to the future or the past. Is this possible, and if so what are the implications for space travel? Let's begin with a simple example.

Example 29

Twins Bib and Bub are separated at birth. Bub is placed in a spacecraft that leaves Earth and travels at a constant speed of $0.75c$. Bib remains on Earth. When Bib reaches his 80th birthday (in Earth years), calculate how old he determines his twin Bub to be.

Solution Since Bub is moving at $0.75c$ relative to Bib, then according to Bib (and using Table 1.7), Bub is now approximately 80×1.5 Earth years old (~ 120!), as measured by Bib's clocks. Conversely, on his 80th birthday, Bub will determine that Bib is now ~ 120 years old (by Bub's clocks). Who is correct? They both are! This appears at first to be nonsense. It has to be remembered, however, that Bib and Bub are in different frames of reference and hence can differ in their answers. There is no reference frame that is any more correct than any other.

The twin paradox

The previous example leads us to one of the celebrated paradoxes of relativity; the twin paradox. According to Bib, Bub will age more slowly. (For every 1.5 of Bib's years, only 1 year will pass for Bub.) But since all motion is relative, then according to Bub, Bib ages more slowly.

If Bib now returns to Earth, will he and Bub agree on who has aged the least? (Remember each thinks the other ages less. Since they are back in the same reference frame they must agree but they both can't be right.)

The paradox is resolved when we remember that the special theory of relativity applies in inertial frames of reference only[17]. But Bub has left the Earth, accelerated to reach $0.75c$, reversed direction to return to Earth and then decelerated to come to rest. Bub is therefore in a non-inertial frame of reference for part of the journey and so the rules do not apply to him.

How to be younger than your twin

The rules of special relativity, however, apply to Bib's observations. Bib sees Bub age slower[18]. At $0.75c$, 1.5 of Bib's years pass for every one of Bub's years. Suppose that 60 years elapse (according to Bib) before Bub returns. Then Bub only ages by $\frac{60}{1.5} = 40$ years. Although born at the same time, Bub is now 20 years younger than Bib!

Time travel

If Bub had travelled at $0.999c$, he would have aged by $\frac{60}{22.37} = 2.68$ years. (Obviously the faster the velocity, the less time elapses.)

Time dilation suggests it may be possible to travel in one lifetime to distant stars. But there is a catch. Suppose you wish to travel to a star 100 light years away at $0.999c$ relative to Earth. At this speed, slightly over 100 Earth years elapse as measured by an observer on Earth. As measured by you in the spacecraft it takes only $\frac{100}{22.37} = 4.47$ years. You could get to your destination but you could not go back to the era you left. Over 200 Earth years would elapse before you could return. Everyone you knew would be dead (unless by then we have found the answer to ageing).

The relativity of time allows for space travel into the future but not into the past. KCq page 50 HSC page 53

17. The General Theory or Relativity applies to non-ionertial frame or reference.
18. The General Theory of Relativity applied to Bub also shows Bub ageing less than Bib.

1.3 The solar system is held together by gravity

25 min

OBJECTIVE-RESPONSE QUESTIONS

1 Two bodies of mass m_1 and m_2 whose centres are separated by a distance d attract each other with a gravitational force of F. If the mass of each body is doubled and their separation reduced to one-quarter of its original value, the new force of attraction is given by:

A F
B $16F$
C $32F$
D $64F$ (1 mark)

2 A satellite of mass m travels with a speed v in a circular path of radius R (between centres) around a planet of mass M. The equation which best describes the motion of the satellite is:

A $\frac{mv^2}{R} = \frac{T^2}{R^3}$ B $\frac{mv^2}{R} = \frac{GM}{R^2}$

C $\frac{mv^2}{R} = \frac{GMm}{R^2}$ D $\frac{GMm}{R^2} = \frac{T^2}{R^3}$ (1 mark)

3 A satellite is placed in a circular orbit around the Earth with constant speed. The height is such that air resistance can be neglected. It is true that:

A the resultant force on the satellite is zero
B gravitational attraction to the Earth provides the centripetal force on the satellite
C the velocity of the satellite is constant
D the acceleration of the satellite is constant.
(1 mark)

4 The slingshot effect is used to assist in the propulsion of spacecraft through space. This effect relies on the law of:

A conservation of mass
B conservation of linear momentum
C conservation of angular momentum
D conservation of charge. (1 mark)

SHORT-ANSWER QUESTIONS

5 Newton once wrote:

I began to think of gravity extending to the orbit of the moon and ... from Kepler's rule (Third Law) ... I deduced that the force which keeps the planets in their orbits must be reciprocally as the square of the distances from the centres about which they revolve: and thereby compared the force requisite to keep the moon in her orbit with the force of gravity at the surface of the Earth and found them to fit pretty nearly.

a Deduce what Law Newton is referring to in this quote. (1 mark)

b Given that the moon is 60 Earth-radii from the Earth's centre and the acceleration due to gravity at the Earth's surface is 9.8 $m.s^{-2}$, calculate the acceleration of the moon towards the Earth.
(2 marks)

6 Two objects of mass M and $2M$ have radii of r and $2r$ respectively.

a Compare their forces of attraction to a 1 kg mass. (2 marks)

b Identify an alternative name for what you have just calculated. (1 mark)

LONGER-ANSWER QUESTIONS

7 The moon has a mass of 7.35×10^{22} kg and orbits the Earth at an average distance of 406 700 km.

a Calculate the gravitational force of attraction between the Earth and moon. (1 mark)

b Identify the magnitude of the centripetal force acting on the moon. (1 mark)

c Calculate the moon's orbital speed. (2 marks)

1.4 Current and emerging understanding about time and space has been dependent upon earlier models of the transmission of light

1 hr 10 min

OBJECTIVE-RESPONSE QUESTIONS

1 According to Galilean-Newtonian relativity, all but one of the following statements is correct. Which is the incorrect statement?

A The laws of mechanics are the same for an observer at rest and one moving at a constant velocity relative to the first.
B The velocity of an observer A relative to another observer B is given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$.
C The length of a moving object depends upon the reference frame from which it is viewed.
D The speed of light c is relative to the aether.
(1 mark)

2 The incorrect statement is:

A Galileo attempted to measure the speed of light but could only conclude it was extremely fast.
B All measured quantities are relative quantities.
C Newton regarded space and time as being dependent.
D The aether was believed to be the medium through which light could propagate. (1 mark)

3 For the Michelson–Morley experiment, which statement is incorrect?

A No motion of the Earth relative to the aether was detected.

B The speed of light depends on the motion of the observer through the aether.

C Interference methods were used to look for motion through the aether.

D No aether wind was detected. (1 mark)

4 If two separated events are simultaneous for an observer, then the same two events will:

A be simultaneous for all other observers

B not be simultaneous for an observer moving with constant velocity relative to the first observer

C be simultaneous only for observers in inertial frames of reference

D be simultaneous for a second observer only if the events are separated by a distance less than one light year. (1 mark)

5 An astronomer measures the speed of recession of a distant galaxy by means of its 'red shift' as $\frac{3c}{4}$. Radio signals coming from the galaxy reach Earth at:

A c. B $\frac{3c}{4}$ C $\frac{c}{4}$ D $\frac{c}{2}$ (1 mark)

6 A metre rule is seen by an observer stationary with respect to the rule, and by a second observer moving at a speed of $\frac{c}{2}$ relative to the rule. The second observer observes the rule to be:

A 1.0 m

B much more than 1.0 m

C slightly more than 1.0 m

D slightly less than 1.0 m. (1 mark)

SHORT-ANSWER QUESTIONS

7 a Identify the purpose of the Michelson-Morley experiment? (1 mark)

b Identify the role of the half-silvered mirror. (1 mark)

c Identify why an interferometer was used in the experiment. (1 mark)

d Recall the result of the experiment. (1 mark)

8 Explain the meaning of the phrase 'the relativity of simultaneity'. (3 marks)

9 Experiments prove that the speed of light is constant and is the same for all observers moving with constant velocity relative to each other. Recall three consequences that result from this fact of nature. (3 marks)

10 By reference to a hypothetical 'light clock', describe why time is dilated (lengthened) for a moving observer when measured by a stationary observer. (3 marks)

11 An unidentified flying object (UFO) is observed by a stationary observer to be 10 m long and travelling at $0.4c$.

a Calculate the length of the UFO as measured by the extraterrestrial pilot. (1 mark)

b Calculate how long an observer on Earth measures as having elapsed if the extraterrestrial pilot measures 1 hour of his time to have passed. (1 mark)

LONGER-ANSWER QUESTIONS

12 This question refers to the Michelson-Morley experiment.

a Recall two reasons put forward prior to Einstein to 'explain' the negative results of the experiment. (2 marks)

b Describe how Einstein explained the result. (2 marks)

c Recall one implication that comes from Einstein's explanation and explain how this occurs. (2 marks)

13 This question refers to relativity.

a Define the meaning of the aether. (1 mark)

b Mu-mesons (muons) are elementary particles that come to Earth in cosmic ray showers. They disintegrate spontaneously after an average lifetime of 2.2×10^{-6} s (in their reference frame). Even allowing for their fast speed ($0.999c$), in this short lifetime they should not be able to travel more than 600 m. The muons, however, are created at the top of the atmosphere some 10 km up and reach the Earth's surface (where they are detected in laboratories). Explain these observations:

i in the reference frame of the muon (1 mark)

ii in the Earth' reference frame. (1 mark)

14 A spaceship passes you at a speed of $0.80c$.

a You measure its length to be 75 m. Calculate the length a member of the spaceships' crew would measure it to be. (2 marks)

c If a member of the crew held a 1 kg mass in his hand, what mass would an observer on the Earth calculate he was holding? (2 marks)

d If the spaceship were to travel to a star 100 light years away from Earth (as measured by an Earth bound observer) at this speed, calculate how far the ship's crew would determine the distance to the star to be. (2 marks)

15 Outline how, using the principles of relativity, it is theoretically possible for astronauts to visit 'nearby' stars in ***their*** lifetime but it is unlikely that any government would fund such a journey. (5 marks)

C is correct because the slingshot effect is due to conservation of angular momentum. [1.3.4]

5 EC You should recognise that the law of universal gravitation is responsible for keeping the moon in orbit around the Earth.

EM 1 mark for correct choice of equation; 1 mark for correct answer.

a The Law of Universal Gravitation. ✓

b The acceleration due to gravity on a planet is given by: $g = \frac{GM}{d^2}$. Hence:

$$\frac{g_1}{g_2} = \frac{d_2^2}{d_1^2}$$

$$g_2 = \frac{d_1^2}{d_2^2} g_1 = \frac{R^2}{(60R)^2} g_1$$

$$= \frac{1}{3600} \times 9.8 = 2.7 \times 10^{-3} \text{ m.s}^{-2}$$

✓ ✓ [1.3.1] [1.1.2]

6 EC The question requires that you calculate the attraction of both masses and compare their ratio. The equation for gravitational field strength is numerically equal to the gravitational force of attraction on a 1 kg mass.

a Comparing the forces we have:

$$F = \frac{Gm_1m_2}{d^2}, \text{ so}$$

$$F_{2M} = \frac{G \times 1 \times (2M)}{r^2} = \frac{2GM}{r^2}$$

$$F_M = \frac{G \times 1 \times (M)}{(2r)^2} = \frac{GM}{4r^2} = \tfrac{1}{8} F_{2M}$$

✓ ✓

b This is essentially the gravitational field strength (or the acceleration due to gravity) ✓ [1.3.1] [1.3.2]

7 EC You need to use the data sheet provided to find the value of G and the mass of the Earth.

EM In **a** 1 mark for correct substitution and 1 mark for correct answer; in **b** 1 mark for recognising that the centripetal force for satellites is provided by the force of universal gravitation; in **c** 1 mark for correct equation for v; 1 mark for correct substitution of data.

a

$$F = \frac{Gm_1m_2}{d^2}$$

$$= \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 5.983 \times 10^{24}}{(406700 \times 10^3)^2}$$

$$= 1.77 \times 10^{20} \text{ N}$$

b The centripetal force is provided by the gravitational attraction between the Earth and moon, i.e. $F_c = 1.77 \times 10^{20}$ N ✓

c
$$F_c = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{F_c r}{m}}$$

$$= \sqrt{\frac{1.77 \times 10^{20} \times 406700 \times 10^3}{7.35 \times 10^{22}}}$$

$$= 990 \text{ m.s}^{-1}$$

✓ ✓ [1.3.2][1.2.7]

1.4

1 **C** This is a statement of Einsteinian relativity, not Newtonian relativity. Answers **A**, **B** and **D** are all true statements but the question asked for the incorrect statement. [1.4.5]

2 **C** Newton regarded time and space as independent. **A**, **B** and **D** are true statements, not incorrect statements as asked for. [1.4.5]

3 **B** The speed of light is constant. This was the null result of the Michelson-Morley experiment. Answers **A**, **C** and **D** are true statements but the question asked for the incorrect statement. [1.4.2] [1.4.6]

4 **B** is the correct statement of the relativity of simultaneity. Answers **A**, **C** and **D** are untrue. [1.4.10]

5 **A** is correct because c is constant. This is the basic premise of special relativity. Velocities of light do not add or subtract as for normal velocities. [1.4.6]

6 **D** is correct as it is an example of length contraction, i.e. the ruler shrinks in the direction of motion. Answers **A**, **B** and **C** are untrue. [1.4.11]

7 EM 1 mark for identifying the purpose of Michelson-Morley experiment. 1 mark for idea of coherence; 1 mark for comment about sensitivity of interference; 1 mark for null result.

a The Michelson-Morley experiment attempted to measure the velocity of the Earth relative to the aether. ✓

b The half-silvered mirror was used to split the light beam to form two coherent beams ✓ so that an interference pattern could be formed to measure motion through the aether.

c The expected change in the speed of light was calculated to be tiny. A sensitive measuring device was needed and an 'interferometer' was used. ✓

d No motion of the Earth relative to the aether was detected. ✓ [1.4.2]

8 EM 1 mark for clear concept of simultaneity; 1 mark for effect of constancy of velocity of light.

Two events that occur at the same time relative to an observer are said to be simultaneous. ✓ That these same two events, however, will not be simultaneous for a second observer moving with constant velocity relative to the first observer ✓ is due to the constancy of the speed of light. [1.4.10]

9 EM 1 mark for each correct implication to a maximum of three marks.

The consequences of the constancy of the speed of light are: (Any three.)

- Time dilation. Time for a moving observer goes slower than for a stationary observer. ✓
- Length contraction. The length of a moving object contracts in the direction of its motion. ✓
- Space-time continuum. Space and time are interdependent. An event requires four dimensions (x, y, z, t) to define it. ✓
- Relativity of simultaneity. ✓
- Mass dilation. The mass of a moving object increases with increasing speed. ✓
- Mass-energy. The equivalence of mass and energy. They are two manifestations of the same entity. ✓ [1.4.6]

10 EC For full marks reference must be made to the different paths 'observed' by different observers and relate this to the speed of light being constant; 1 mark for correct substitution or data; 1 mark for correct answer.

Refer to Figure 1.31 in the text. Relative to an observer travelling on the train the light beam travels up and down. ✓ To a stationary observer however, the light beam follows a zigzag path and so travels further. ✓ Because the speed of light is constant it follows that the time must be 'lengthened' ✓ (so the ratio of distance to time is constant). [1.4.9] [1.4.12]

11 EM 1 mark for correct substitution of data; 1 mark for correct answer.

a The length of a moving object is given by
$$l_v = l_o\sqrt{1 - \frac{v^2}{c^2}}$$
where l_o is the length relative to a stationary observer and l_v is the length for an observer moving with the object. Hence:

$$l_v = l_o\sqrt{1 - \frac{v^2}{c^2}}$$

$$l_o = \frac{l_v}{\sqrt{1 - \frac{v^2}{c^2}}} \quad ✓$$

$$= \frac{10}{\sqrt{1 - \frac{(0.4c)^2}{c^2}}}$$

$$= 10.91 \text{ m}$$

b The time in a moving frame is given by:

$$t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1.0}{\sqrt{1 - \frac{(0.4c)^2}{c^2}}} \quad ✓$$

$$= 1.09 \text{ h}$$

$$= 1 \text{ h } 5.4 \text{ min}$$

[**PROBLEM SOLVING** Length contraction]
[**PROBLEM SOLVING** Time dilation]

12 EM In **a** 1 mark for each correct statement to a maximum of 2 marks; in **b** 1 mark for a statement of the constancy of the speed of light. See Question 9 above.

a Two reasons put forward to explain the null result of the Michelson-Morley experiment were:
- Length contraction had occurred in the arm parallel to the direction of the motion of the apparatus. ✓
- The aether was being 'dragged along' by the Earth. ✓

b Einstein explained the negative result by stating that the speed of light is constant ✓ and is unaffected by the velocity of the source of the observer. ✓

c See answer to Question 9 for implications from this law. [1.4.5] [1.4.6]

13 EM 1 mark for correct definition of the aether.

EC Experiments such as this have provided conclusive evidence supporting the Theory of Special Relativity.

a The aether was proposed to be the medium through which light waves could propagate. ✓

b i The muon 'sees' the atmosphere contracted ✓ in its direction of motion to an extent such that it can traverse the shorter distance before it disintegrates.

ii Earth observers see the muon's lifetime dilated ✓ by the factor $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ allowing it to last long enough to reach the Earth's surface. [1.4.1] [1.4.11] [1.4.12]

14 EM 1 mark in each for correct substitution of data; 1 mark in each for correct answer.

a $l_v = l_o\sqrt{1 - \frac{v^2}{c^2}}$

$$75 = l_o\sqrt{1 - \frac{(0.8c)^2}{c^2}}$$

$$= l_o \times 0.6$$

$$l_o = 125 \text{ m} \quad ✓$$

b Mass dilation is given by:

$$m_v = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}}$$

$$= \frac{1}{\sqrt{0.36}}$$

$$= 1.67 \text{ kg} \quad ✓$$

c We need to find t_o in the time dilation formula:

$$t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}} \quad ✓$$

$$t_o = 100 \times 0.6 = 60 \text{ light years}$$

[**PROBLEM SOLVING** Length contraction]
[**PROBLEM SOLVING** Mass dilation]

15 EM Five marks for clear understanding of the idea of time dilation and the different times that would elapse for the Earth observer and the spaceship crew, and that the implications make funding by any contemporary government unlikely, as all its members would be long dead and buried before the spaceship could return; 3-4 marks some understanding of the concept of time dilation; 1-2 marks for a little understanding.

One of the consequences of Special Relativity is that time dilates, ✓ that is, time for a moving observer goes slower relative to a stationary observer. Suppose a spacecraft was sent to a star 100 light-years away. At the enormous speed of 0.999c, this would take slightly over 100 years *relative to a stationary observer on Earth*. ✓ As measured by the crew of the spacecraft, however, it would take much less time (~4.47 years). ✓ Although they could return to Earth in around 10 years of their time, ✓ over two centuries would have passed on Earth. Government are unlikely to invest in such an expedition! ✓ [1.4.15]

Chapter 2: Motors and generators

KCq Key Concept questions (pages 55–88)

Note: lower case i will be used for current in this section to distinguish it from l for length.

1 There are a number of alternative experiments you could describe. Your description should comment on the need for permanent magnets and a current-carrying conductor that is free to move. The forces are generally small and so the setup has to be sensitive.

2 You should use the currents available from a school transformer/power supply. You must not use 240 V directly as it is dangerous.

3 The motor effect is where a force acts on a current-carrying conductor in a magnetic field.

4 Right hand palm rule: if the fingers of the right hand point in the direction of the magnetic field and the thumb points in the direction of the conventional current, then the palm points in the direction of the force.

5 a Up the page.
b To the left of the page

6 Factors include:
a length of conductor
b strength of magnetic field
c amount of current in conductor
d angle between conductor and field.

7 The force is actually on the electrons in the conductor so: the longer the conductor the more electrons experience the force simultaneously and hence the greater the force; the stronger the field the stronger the force on each electron; the more current the more electrons and hence the bigger the force; the closer the angle between the conductor and magnetic field is to 90° the bigger the force.

8 a Force on the current-carrying wire is given by:

$$F = Bil\sin\theta$$
$$= 10 \times 5 \times 1.5 \times \sin 0$$
$$= 0 \text{ N}$$
$$F = Bil\sin\theta$$
$$= 10 \times 5 \times 1.5 \times \sin 60$$
$$= 65 \text{ N}$$
$$F = Bil\sin\theta$$
$$= 10 \times 5 \times 1.5 \times \sin 90$$
$$= 75 \text{ N}$$