The cubic meter is the unit used for measuring ordinary solids, such as excavations or embankments (see Table 5-20).

Table 5-20 Metric Table of Cubic Measure

Measure	*Equivalent*	*Equivalent*
1000 cubic millimeters (mm^3)	1 cubic centimeter	0.061 + cubic inches
1000 cubic centimeters (cm^3)	1 cubic decimeter	61.026 + cubic inches
1000 cubic decimeters (dm^3)	1 cubic meter	35.316 + cubic feet

The liter is the unit of capacity, both of liquid and of dry measures, and is equivalent to a vessel whose volume is equal to a cube whose edge is $^1/_{10}$ of a meter, equal to 1.0567 quarts liquid measure, and 0.9081 quart dry measure (see Table 5-21).

The hectoliter is the unit used for measuring liquids, grain, fruit, and roots in large quantities. The gram is the unit of weight equal to the weight of a cube of distilled water, the edge of which is $^1/_{100}$ of a meter, and is equal to 15.432 troy grains (see Table 5-22).

Geometry

By definition, *geometry* is that branch of mathematics that deals with space and figures in space. In other words, it is the science of the mutual relations of points, lines, angles, surfaces, and solids that are considered as having no properties except those arising from extension and difference of situation.

Lines

The two kinds of lines are straight and curved. A *straight line* is the shortest distance between two points. A *curved line* is one that changes its direction at every point. Two lines are said to be parallel when they have the same direction. A horizontal line is one parallel to the horizon or surface of the Earth. A line is perpendicular with another line when they are at right angles to each other. These definitions are illustrated in Figure 5-9.

Angles

An *angle* is the difference in direction between two lines proceeding from the same point (called the *vertex*). Angles are said to be *right* (90 degrees) when formed by two perpendicular lines (see

Table 5-21 Metric Table of Capacity

10 milliliters (ml.)	= 1 centiliter	= .0338 fluid ounce
10 centiliters (cl.)	= 1 deciliter	= .1025 cubic inch
10 deciliters (dl.)	= 1 liter	= 1.0567 liquid quart
10 liters (l.)	= 1 dekaliter	= 2.64 gallons
10 dekaliters (dl.)	= 1 hectoliter	= 26.418 gallons
10 hectoliters (hl.)	= 1 kiloliter	= 264.18 gallons
10 kiloliters (kl.)	= 1 myrialiter (ml.)	
1 myrialiter	= 10 cubic meters	
	= 283.72 + bushels	= 2641.7 + gallons
1 kiloliter	= 1 cubic meter	
	= 28.372 + bushels	= 264.17 gallons
1 hectoliter	= $^1/_{10}$ cubic meter	
	= 2.8372 + bushels	= 26.417 gallons
1 decaliter	= 10 cubic decimeters	
	= 9.08 quarts	= 2.6417 gallons
1 liter	= 1 cubic decimeter	
	= .908 quart	= 1.0567 quart liquid
1 deciliter	= $^1/_{10}$ cubic decimeter	
	= 6.1022 cubic inches	= .845 gallons
1 milliliter	= 10 cubic centimeters	
	= .6102 cubic inches	= .338 fluid ounces
1 centiliter	= 1 cubic centimeter	
	= .061 cubic inches	= .27 fluid dram

Figure 5-10A), *acute* (less than 90 degrees) when less than a right angle (see Figure 5-10B), and *obtuse* (more than 90 degrees) when greater than a right angle (see Figure 5-10C). All angles except right (or 90-degree) angles are called *oblique angles*.

Angles are usually measured in degrees (circular measure) (see Figure 5-10D). The *complement* of an angle is the difference between 90 degrees and the angle. The *supplement* of the angle is the difference between the angle and 180 degrees.

Plane Figures

The term *plane figures* means a plane surface bounded by straight or curved lines, and a *plane* (or *plane surface*) is one in which any straight line joining any two points lies wholly in the surface. Figure 5-11 defines a plane surface. There is a great variety of plane

Table 5-22 Metric Table of Weight Measure

Measure	*Equivalent*	*Equivalent*
10 milligrams (mg)	1 centigram	0.15432 + grains troy
10 centigrams (cg)	1 decigram	1.54324 + grains troy
10 decigrams (dg)	1 gram	15.43248 + grains troy
10 grams (g)	1 dekagram	0.35273 + ounce avoirdupois
10 dekagrams (Dg)	1 hectogram	3.52739 + ounces avoirdupois
10 hectograms (hg)	1 kilogram	2.20462 + pounds avoirdupois
10 kilograms (kg)	1 myriagram	22.04621 + pounds avoirdupois
10 myriagrams (Mg)	1 quintal	220.46212 + pounds avoirdupois
10 quintals	1 ton	2204.62125 + pounds avoirdupois

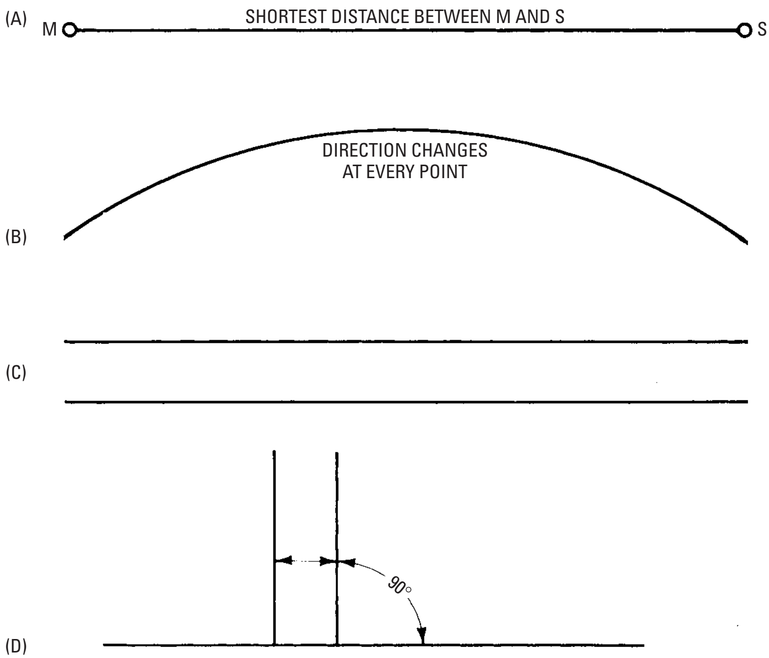


Figure 5-9 Various lines: (A) straight, (B) curved, (C) parallel, and (D) perpendicular.

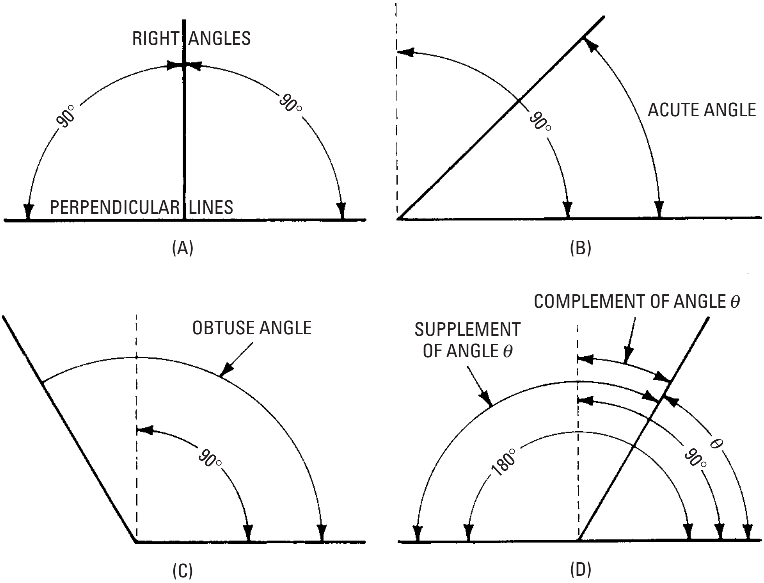


Figure 5-10 Various angles: (A) right, (B) acute, (C) obtuse, and (D) complement and supplement of an angle.

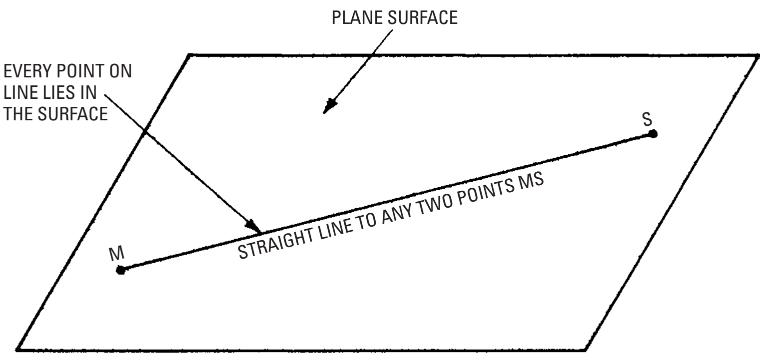


Figure 5-11 A plane surface means that every point on a straight line joining any two points in the surface lies in the surface.

figures, which are known as *polygons* when their sides are straight lines. The sum of the sides is called the *perimeter*. A regular polygon has all its sides and angles equal. Plane figures of three sides are known as triangles (see Figure 5-12), and plane figures of four sides are quadrilaterals. Figure 5-13 shows examples of these. Various plane figures are formed by curved sides and are known as circles, ellipses, and so forth, as shown in Figure 5-14. Figure 5-15 details the structure of the quadrilateral.

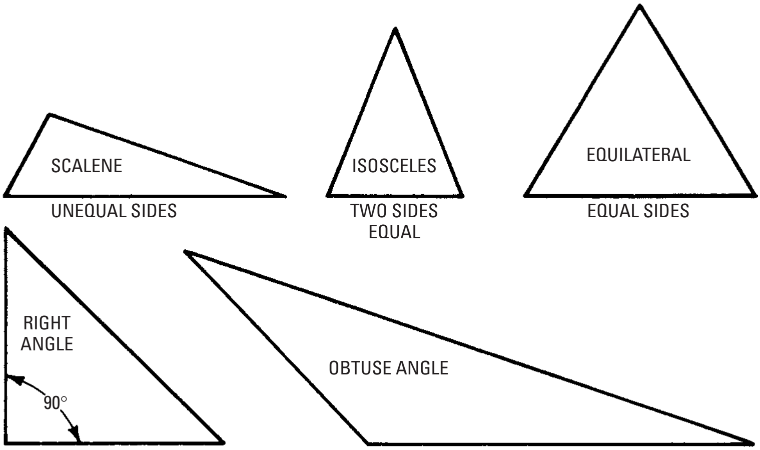


Figure 5-12 Various triangles. A triangle is a polygon having three sides and three angles.

Solids

Solids have three dimensions—length, width, and thickness. The bounding planes are called the *faces*, and the intersections are called the *edges*. A prism (see Figure 5-16) is a solid whose ends consist of equal and parallel polygons, and whose sides are *parallelograms*. The *altitude* of a prism is the perpendicular distance of its opposite sides or bases. A *parallelopipedon* is a prism that is bounded by six parallelograms; the opposite parallelograms are parallel and equal. A *cube* is a parallelopipedon whose faces are equal. One important solid is the *cylinder*, which is a body bounded by a uniformly curved surface and having its ends equal and forming parallel circles (see Figure 5-17). There are numerous other solids having curved surfaces (such as *cones* and *spheres*).

Geometrical Problems

The following problems illustrate the method in which various geometrical figures are constructed, and they should be solved by the

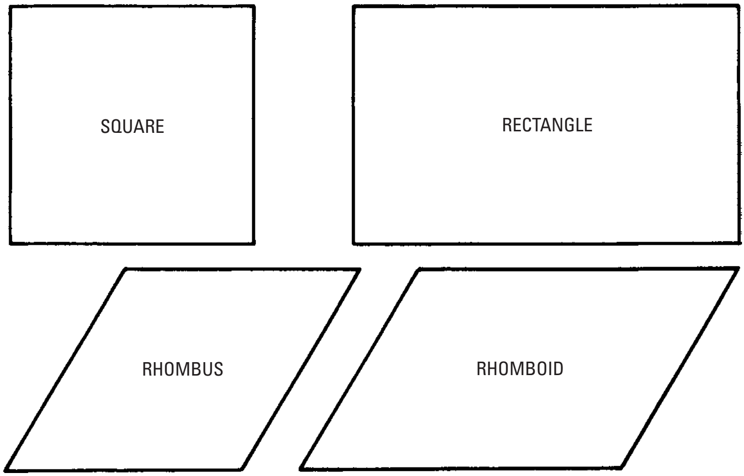


Figure 5-13 Various quadrilaterals. All opposite sides of a quadrilateral are equal.

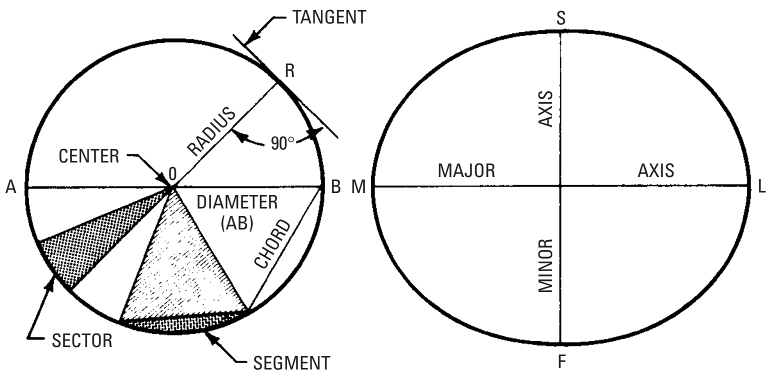


Figure 5-14 Curved figures. A circle is a plane figure bounded by a uniformly curved line, every point of which is equidistant from the center point O. OR is a radius, and AB is a diameter. An ellipse is a curved figure enclosed by a curved line that is such that the sum of the distances between any point on the circumference and the two foci is invariable. ML is the major axis, and SF is the minor axis.

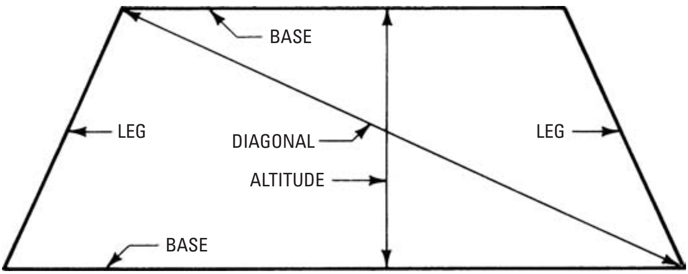


Figure 5-15 The parallel sides of a quadrilateral (four-sided polygon) are the bases. The distance between the bases is the altitude, and a line joining two opposite vertices is a diagonal.

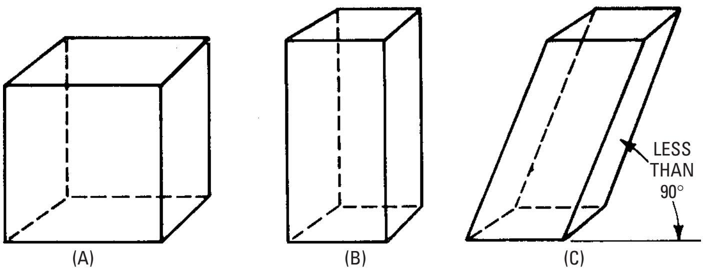


Figure 5-16 Various prisms: (A) cube, or equilateral parallelopipedon; (B) parallelopipedon; and (C) oblique parallelopipedon.

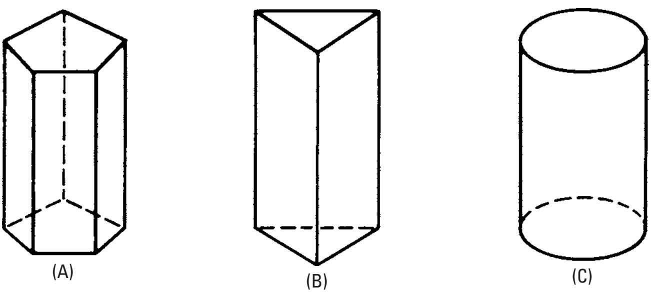


Figure 5-17 Various solids: (A) pentagonal prism, (B) triangular prism, and (C) cylinder.

use of pencil, dividers, compass, and scale. Many of these problems are commonly encountered in carpentry with layout work. Therefore, experience in working them out will be of value to carpenters and woodworkers.

Problem 1

To bisect (or divide into two equal parts) a straight line or arc of a circle.

In Figure 5-18, from the ends A and B, as centers, describe arcs cutting each other at C and D, and draw line CD, which cuts the line at E, or the arc at F.

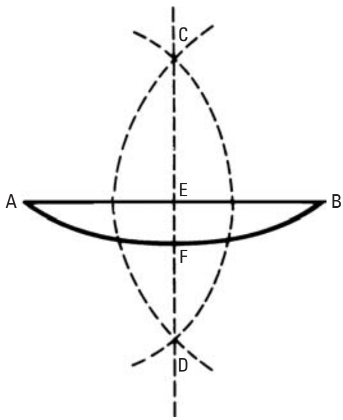


Figure 5-18 To bisect a straight line or arc of a circle.

Problem 2

To draw a perpendicular to a straight line, or a radial line to an arc.

The line CD is perpendicular to AB; also, the line CD is radial to the arc AB (see Figure 5-18).

Problem 3

To erect a perpendicular to a straight line from a given point in that line.

In Figure 5-19, with any radius from any given point A, in the line BC describe arcs cutting the line at B and C. Next, with a longer radius describe arcs with B and C as centers, intersecting at D, and draw the perpendicular DA.

Second Method

In Figure 5-20, from any point F above BC, describe a circle passing through the given point A and cutting the given line at D. Draw DF, and extend it to cut the circle at E. Draw the perpendicular AE.

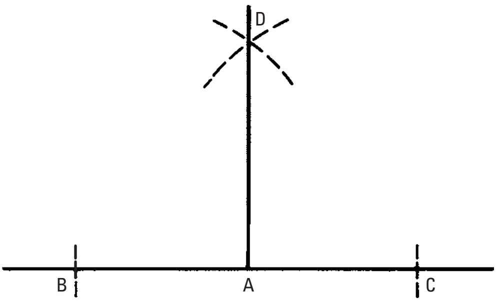


Figure 5-19 To erect a perpendicular to a straight line from a given point on that line.

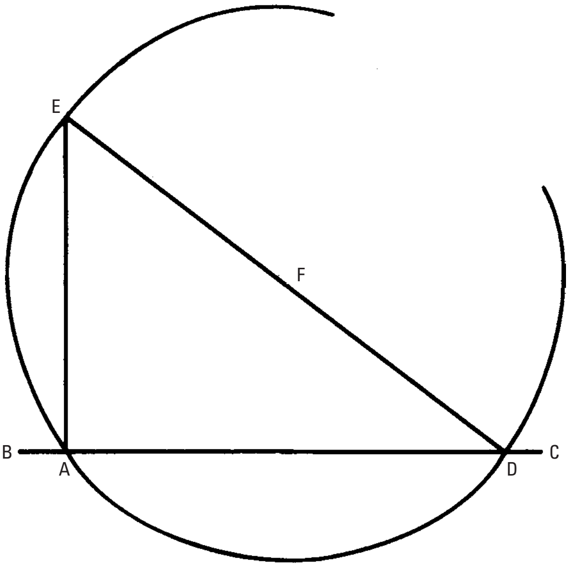


Figure 5-20 To erect a perpendicular to a straight line from a given point on that line, second method.

Third Method (Boat Builders' Layout Method)

In Figure 5-21, let *MS* be the given line and *A* be the given point. From *A*, measure off a distance *AB* (4 feet). With centers *A* and *B* and radii of 3 and 5 feet, respectively, describe arcs *L* and *F* intersecting at *C*. Draw a line through *A* and *C*, which will be the

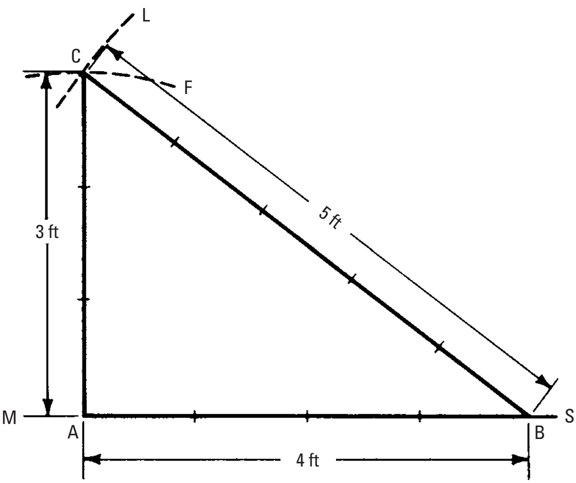


Figure 5-21 To erect a perpendicular to a straight line from a given point on that line, third method.

perpendicular required. This method is used extensively by carpenters when squaring the corners of buildings, but they ordinarily use multiples of 3, 4, and 5 (such as 6, 8, and 10, or 12, 16, and 20).

Fourth Method

In Figure 5-22, from *A*, describe an arc *EC*, and from *E* with the same radius describe the arc *AC*, cutting the other at *C*. Through *C*,

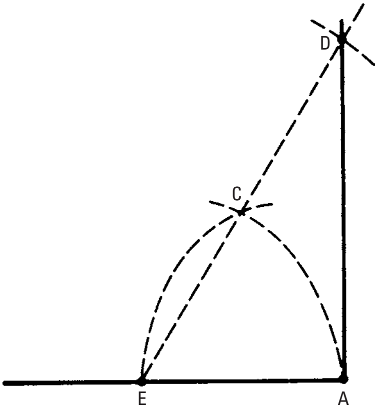


Figure 5-22 To erect a perpendicular to a straight line from a given point on that line, fourth method.

draw a line *ECD*. Lay off *CD* equal to *CE*, and through *D*, draw the perpendicular *AD*. The triangle produced is exactly 60 degrees at *E*, 30 degrees at *D*, and 90 degrees at *A*. The hypotenuse *ED* is exactly twice the length of the base *EA*.

Problem 4

To erect a perpendicular to a straight line from any point outside the line.

In Figure 5-23, from the point *A*, with a sufficient radius cut the given line at *F* and *G*, and from these points describe arcs cutting at *E*. Draw the perpendicular *AE*.

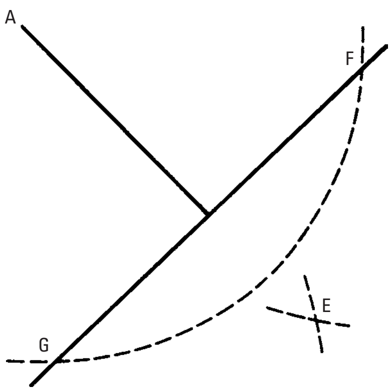


Figure 5-23 To erect a perpendicular to a straight line from any point outside the line.

Second Method

In Figure 5-24, from any two points *B* and *C* at some distance apart in the given line and with the radii *BA* and *CA*, respectively, describe arcs cutting at *A* and *D*. Draw the perpendicular *AD*.

Problem 5

To draw a line parallel to a given line through a given point.

In Figure 5-25, with *C* as the center, describe an arc tangent to the given line *AB*. The radius will then equal the distance from the given point to the given line. Take a point *B* on the given line remote from *C*, and describe an arc. Draw a line through *C*, tangent to this arc at *D*, and it will be parallel to the given line *AB*.

Second Method

In Figure 5-26, from *A*, the given point, describe the arc *FD*, cutting the given line at *F*; from *F*, with the same radius, describe the arc

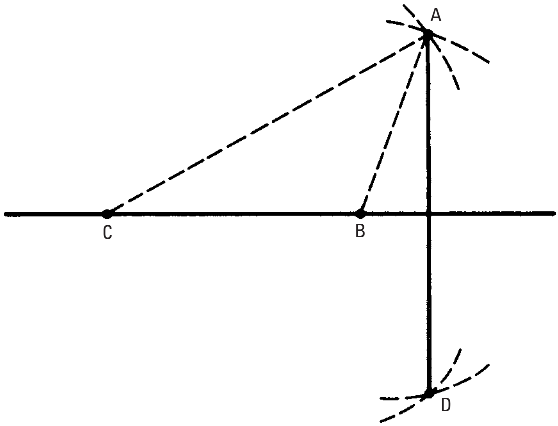


Figure 5-24 **To erect a perpendicular to a straight line from any point outside the line, second method.**

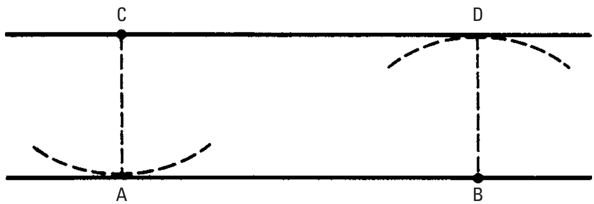


Figure 5-25 **To draw a line parallel to a given line through a given point.**

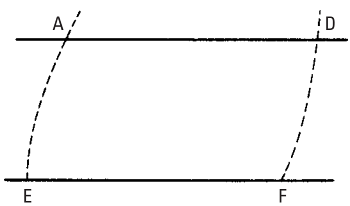


Figure 5-26 **To draw a line parallel to a given line through a given point, second method.**

EA, and lay off *FD* equal to *EA*. Draw the parallel line through the points *AD*.

Problem 6
To divide a line into a number of equal parts.

In Figure 5-27, assuming line *AB* is to be divided into five equal parts, draw a diagonal line *AC* of five units in length. Join *BC* at 5 and through the points 1, 2, 3, and 4. Draw lines 1*L*, 2*a*, and so forth, parallel to *BC*. *AC* will then be divided into five equal parts, *AL, La, ar, rf*, and *fB*.

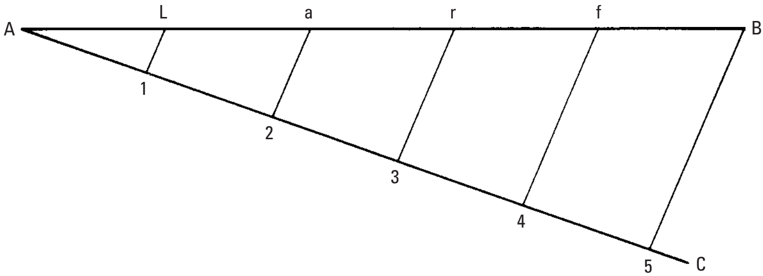


Figure 5-27 To divide a line into a number of equal parts.

Problem 7

To draw an angle equal to a given angle on a straight line.

In Figure 5-28, let *A* be the given angle, and *FG* the line. With any radius from the points *A* and *F*, describe arcs *DE* and *IH* cutting the sides of angle *A* and line *FG*. Lay off arc *IH* equal to arc *DE*, and draw line *FH*. Angle *F* is then equal to *A*, as required

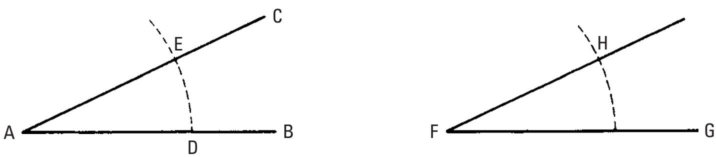


Figure 5-28 To draw an angle equal to a given angle on a straight line.

Problem 8

To bisect an angle

In Figure 5-29, let *ACB* be the angle. With the center of the angle at *C*, describe an arc cutting the sides at *A* and *B*. Using *A* and *B* as centers, describe arcs that intersect at *D*. A line through *C* and *D* will divide the angle into two equal parts.

Problem 9

To find the center of a circle.

In Figure 5-30, draw any chord *MS*. With *M* and *S* as centers, and with any radius, describe arcs *LF* and *L′F′*, and draw a line through their intersection, giving a diameter *AB*. Applying the same construction with centers *A* and *B*, describe arcs *ef* and *e′f′*. A line drawn through the intersections of these arcs will cut line *AB* at *O*, the center of the circle.

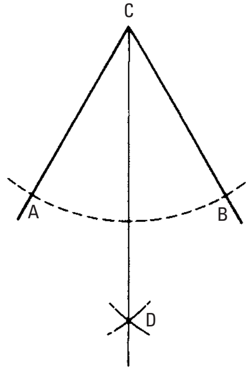


Figure 5-29 To bisect an angle.

Problem 10

To describe an arc of a circle with a given radius through two given points.

In Figure 5-31, take the given points *A* and *B* as centers, and, with the given radius, describe arcs that intersect at *C*. From *C*, with the same radius, describe an arc *AB*, as required.

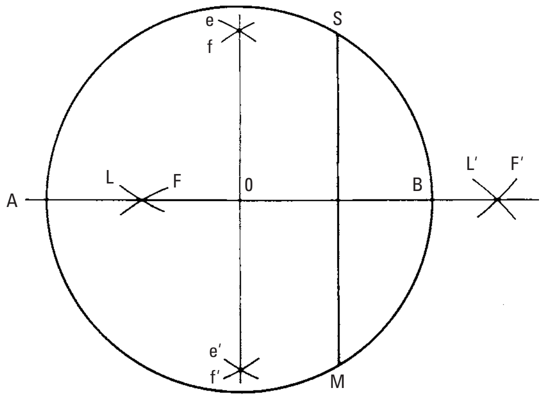


Figure 5-30 Find the center of a circle.

Second Method

In Figure 5-32, for a circle or an arc, select three points *ABC* in the circumference that are well apart. With the same given radius, describe arcs from these three points that intersect each other, and draw two lines, *DE* and *FG*, through their intersections. The point where these lines intersect is the center of the circle or arc.

Problem 11

To describe a circle passing through three given points.

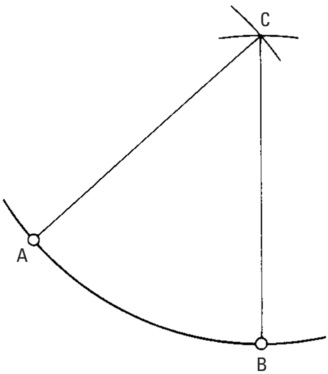


Figure 5-31 To describe an arc of a circle with a given radius through two given points.

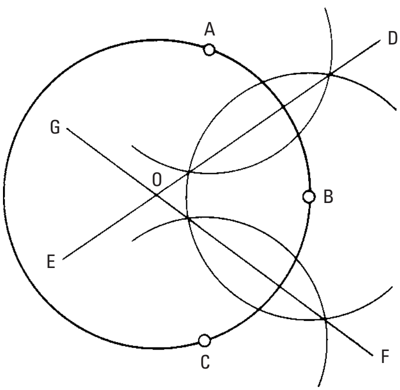


Figure 5-32 To describe an arc of a circle with a given radius through two given points, second method.

In Figure 5-32, let *A*, *B*, and *C* be the given points, and proceed as in Problem 10 to find the center *O* from which the circle may be described. This problem is useful in such work as laying out an object of large diameter (such as an arch) when the span and rise are given.

Problem 12

To draw a tangent to a circle from a given point in the circumference.

In Figure 5-33, from *A*, lay off equal segments *AB* and *AD*. Join line *BD*, and draw line *AE* parallel to *BD* for the tangent.

Problem 13

To draw tangents to a circle from points outside the circle.

In Figure 5-34, from *A*, and with the radius *AC*, describe an arc *BCD*. From *C*, with a radius equal to the diameter of the circle,

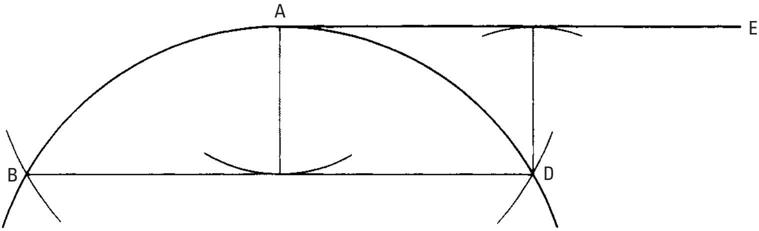


Figure 5-33 To draw a tangent to a circle from a given point in the circumference.

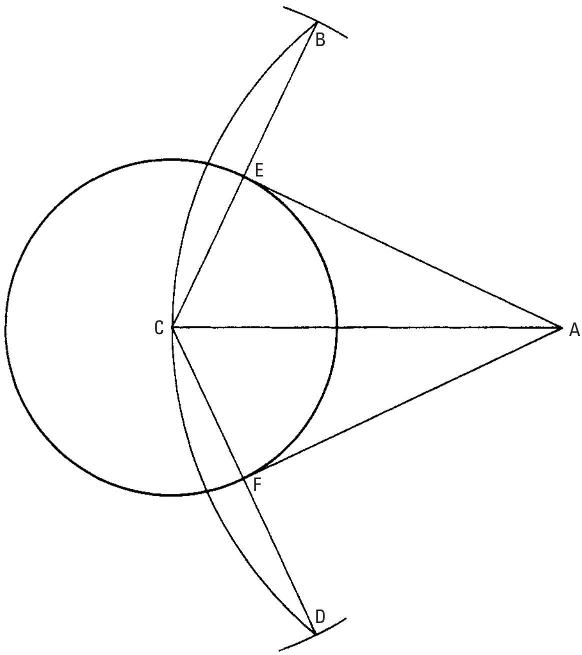


Figure 5-34 To draw tangents to a circle from points outside the circle.

intersect the arc at *BD*. Join *BC* and *CD*, which intersect the circle at *E* and *F*, and draw the tangents *AE* and *AF*.

Problem 14

To describe a series of circles tangent to two inclined lines and tangent to each other.

In Figure 5-35, bisect the inclination of the given lines *AB* and *CD* by the line *NO*. From a point *P* in this line, draw the perpendicular *PB* to the line *AB*, and on *P*, describe the circle *BD*, touching the lines and the centerline at *E*. From *E*, draw *EF* perpendicular to the center line intersecting *AB* at *F*, and from *F*, describe an arc *EG* intersecting *AB* at *G*. Draw *GH* parallel to *BP*, thus producing *H*, the center of the next circle, to be described with the radius *HE*, and so on for the next circle *IN*.

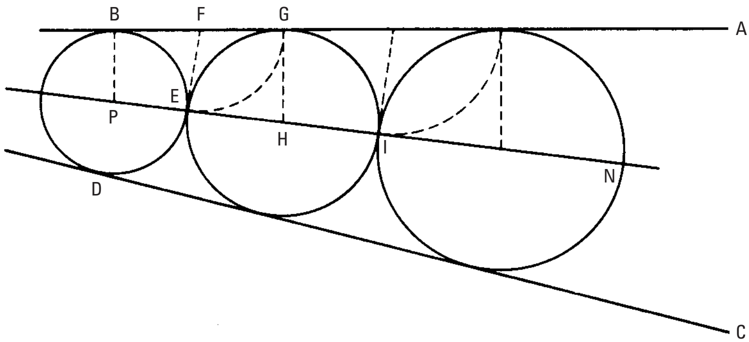


Figure 5-35 To describe a series of circles tangent to two inclined lines and tangent to each other.

Problem 15

To construct an equilateral triangle on a given base.

In Figure 5-36, with *A* and *B* as centers and a radius equal to *AB*, describe arcs *l* and *f*. At their intersection *C*, draw lines *CA* and *CB*, which are the sides of the required triangle. If the sides are to be unequal, the process is the same, taking as the radii the lengths of the two sides to be drawn.

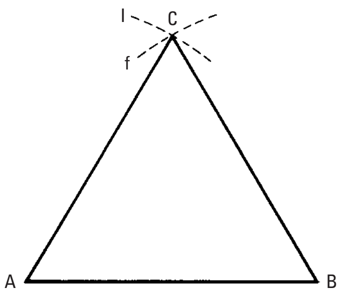


Figure 5-36 To construct an equilateral triangle on a given base.

Problem 16

To construct a square on a given base.

In Figure 5-37, with end points *A* and *B* of the base as centers and a radius equal to *AB*, describe arcs that intersect at *C*; on *C*, describe arcs that intersect the others at *D* and *E*, and on *D* and *E*, intersect these arcs *F* and *G*. Draw *AE* and *BG*, and join the intersections *HI* to form the square *AHIB*.

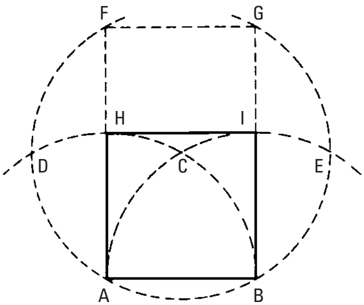


Figure 5-37 To construct a square or a rectangle on a given base.

Problem 17

To construct a rectangle on a given base.

In Figure 5-37, let *AB* be the given base. Erect a perpendicular at *A* and at *B* that is equal to the altitude of the rectangle, and join their ends *F* and *G* by line *FG*. *AFGB* is the rectangle required.

Problem 18

To construct a parallelogram given the sides and an angle.

In Figure 5-38, draw side *DE* equal to the given length *A*, and lay off the other side *DF*, equal to the other length *B*, thus forming the given angle *C*. From *E*, with *DF* as the radius, describe an arc, and from *F*, with the radius *DE*, intersect the arc at *G*. Draw *FG* and *EG*. The remaining sides may also be drawn as parallels to *DE* and *DF*.

Problem 19

To draw a circle around a triangle.

In Figure 5-39, bisect two sides *AB* and *AC* of the triangle at *E* and *F*, and from these points draw perpendiculars intersecting at *K*. From *K*, with radius *KA* or *KC*, describe the circle *ABC*.

Problem 20

To circumscribe and inscribe a circle about a square.

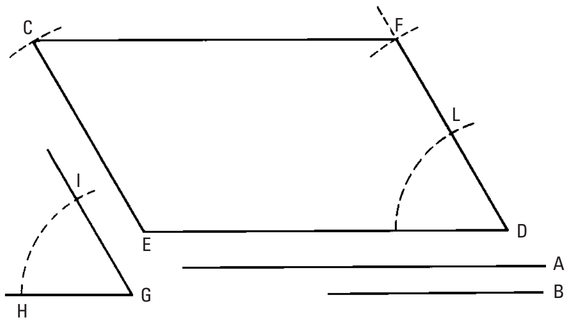


Figure 5-38 To construct a parallelogram given the sides and an angle.

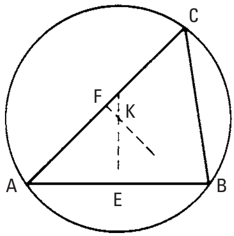


Figure 5-39 To draw a circle around a triangle.

In Figure 5-40, draw the diagonals *AB* and *CD* intersecting at *E*. With a radius *EA*, circumscribe the circle. To inscribe a circle, draw a perpendicular from the center (as just found) to one side of the square, as line *OM*. With radius *OM*, inscribe the circle.

Problem 21

To circumscribe a square around a circle.

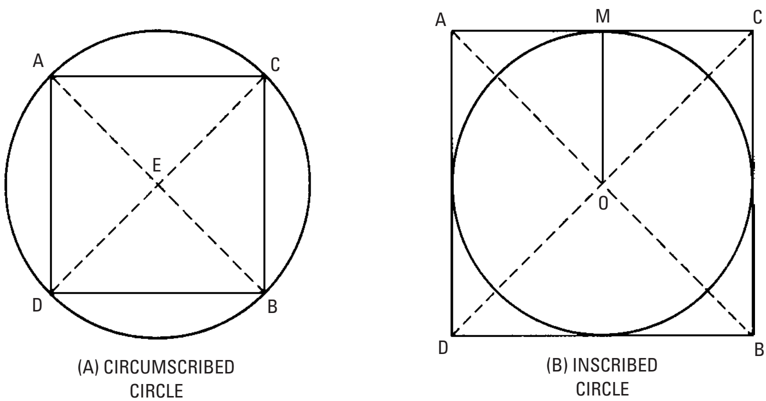


Figure 5-40 (A) To circumscribe a circle around a square. (B) To inscribe a circle inside a square.

In Figure 5-41, draw diameters *MS* and *LF* at right angles to each other. At points *M*, *L*, *S*, and *F*, where these diameters intersect the circle, draw tangents (that is, lines perpendicular to the diameters), obtaining the sides of the circumscribed square *ABCD*.

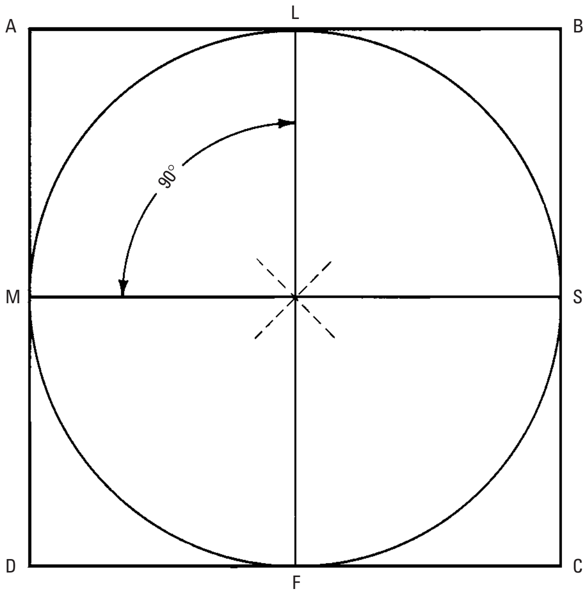


Figure 5-41 To circumscribe a square about a circle.

Problem 22

To inscribe a circle in a triangle.

In Figure 5-42, bisect two angles *A* and *C* of the triangle with lines that intersect at *D*. From *D*, draw a perpendicular *DE* to any side. With *DE* as the radius, describe a circle.

Problem 23

To inscribe a pentagon in a circle.

In Figure 5-43, draw two diameters *AC* and *BD* at right angles intersecting at *O*. Bisect *AO* at *E*, and from *E*, with radius *EB*, *AC* at *F*; from *B*, with radius *BF*. Intersect the circumference at *G* and *H*, and with the same radius, step round the circle to *I* and *K*; join the points thus found to form the pentagon *BGIKH*.

Problem 24

To inscribe a five-pointed star in a circle.

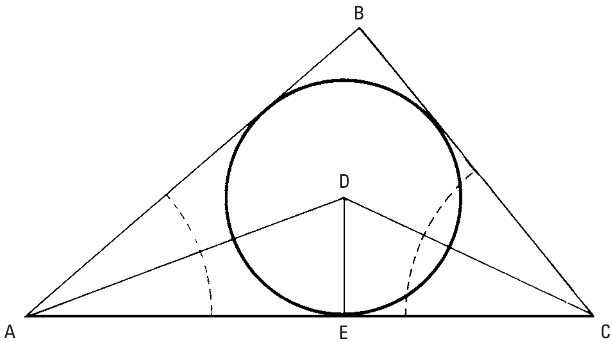


Figure 5-42 To inscribe a circle in a triangle.

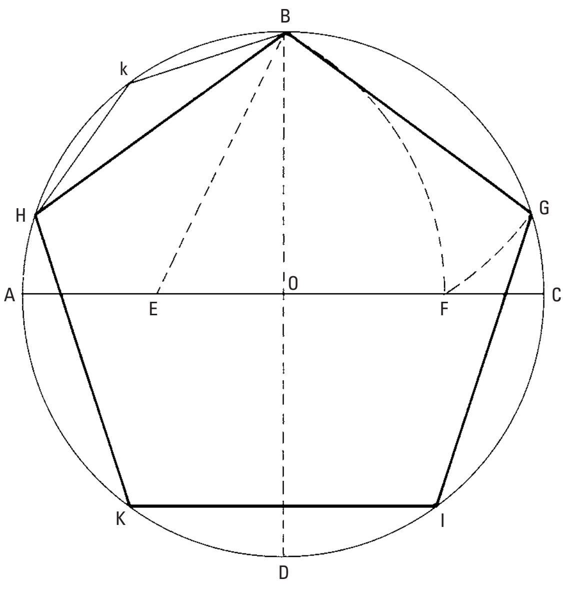


Figure 5-43 To inscribe a pentagon in a circle.

In Figure 5-44, proceed as explained for the inscribed pentagon in Problem 23. Then, connect point *B* with points *K* and *I*, point *H* with points *G* and *I*, and so forth. The star is mathematically correct.

Problem 25

To construct a hexagon from a given straight line

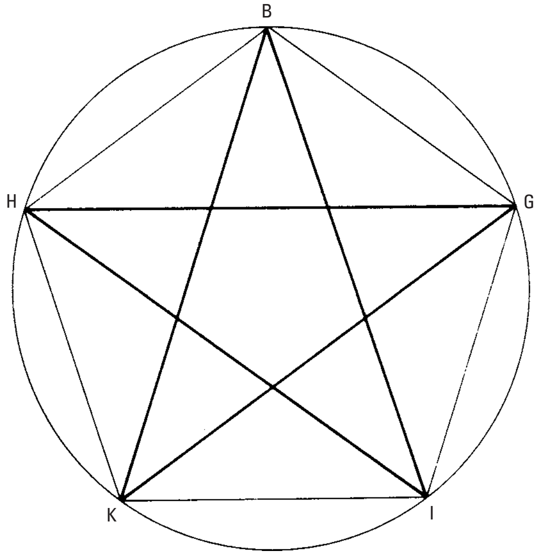


Figure 5-44 To inscribe a five-pointed star in a circle.

In Figure 5-45, from *A* and *B*, the ends of the given line, describe arcs intersecting at *g*. From *g*, with the radius *gA*, describe a circle. With the same radius, lay off arcs *AG*, *GF*, *BD*, and *DE*. Join the points thus found to form the hexagon.

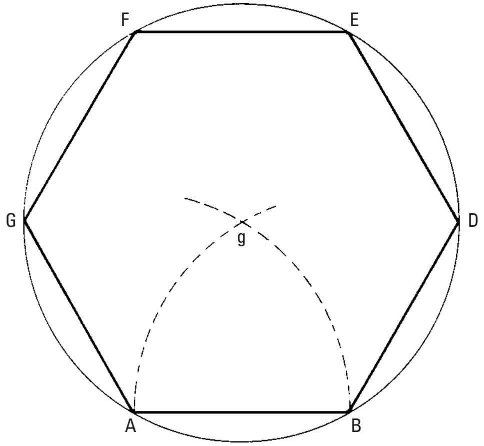


Figure 5-45 To construct a hexagon from a given straight line.

Problem 26

To inscribe a hexagon in a circle.

In Figure 5-46, draw a diameter *ACB*. From *A* and *B*, as centers with the radius of the circle *AC*, intersect the circumference at *D*, *E*, *F*, and *G*, and draw lines *AD*, *DE*, and so forth, to form the hexagon. The points *D*, *E*, and so forth, may also be found by stepping off the radius (with the dividers) six times around the circle.

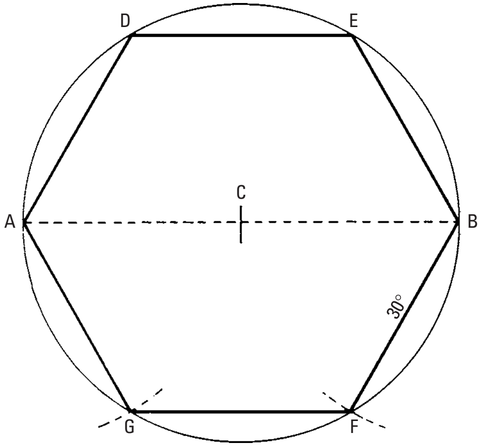


Figure 5-46 To inscribe a hexagon in a circle.

Problem 27

To describe an octagon on a given straight line.

In Figure 5-47, extend the given line *AB* both ways. Now, draw perpendiculars *AE* and *BF*. Bisect the external angles *A* and *B* by using lines *AH* and *BC*. These are made equal to line *AB*. Draw *CD* and *HG* parallel to *AE* and equal to line *AB*. Draw *CD* and *HG* parallel to *AE* and equal to line *AB*. With *G* and *D* as centers, and with the radius equal to *AB*, intersect the perpendiculars at *E* and *F*, and draw line *EF* to complete the hexagon.

Problem 28

To inscribe an octagon in a square.

In Figure 5-48, draw the diagonals of the square intersecting at *e*. From the corners *A*, *B*, *C*, and *D*, with *Ae* as the radius, describe arcs intersecting the sides of the square at *g*, *h*, and so forth, and join the points found to complete the octagon.

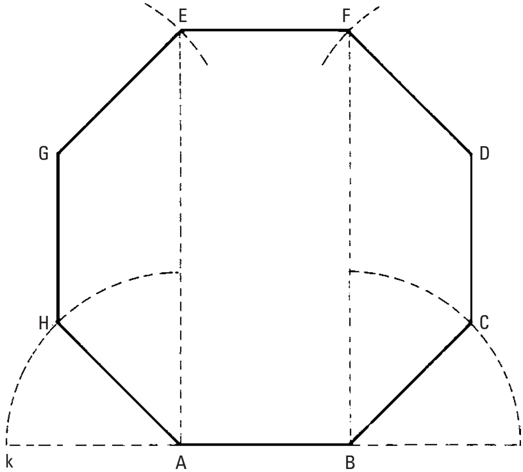


Figure 5-47 To describe an octagon on a given straight line.

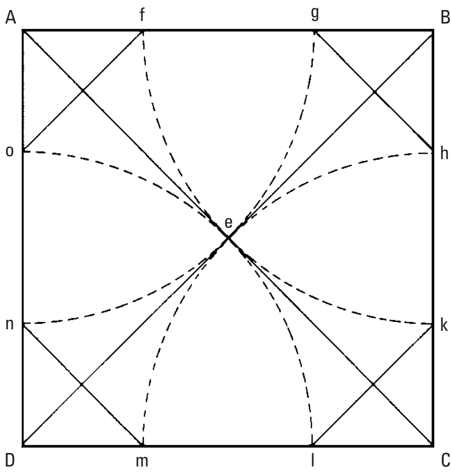


Figure 5-48 To inscribe an octagon in a square.

Problem 29

To inscribe an octagon in a circle.

In Figure 5-49, draw two diameters *AC* and *BD* at right angles. Bisect the arcs *AB*, *BC*, and so forth, at *e*, *f*, and so forth, to form the octagon.

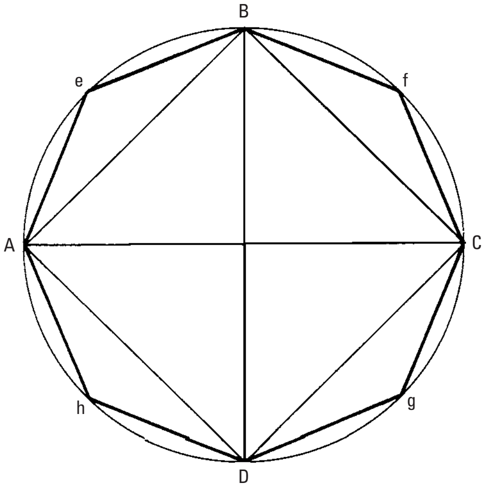


Figure 5-49 To inscribe an octagon in a circle.

Problem 30

To circumscribe an octagon about a circle.

In Figure 5-50, describe a square about the given circle *AB*. Draw perpendiculars *h*, *k*, and so forth, to the diagonals, touching the circle, to form the octagon. The points *h*, *k*, and so forth, may be found by cutting the sides from the corners.

Problem 31

To describe an ellipse when the two axes are given.

In Figure 5-51, draw the major and minor axes *AB* and *CD*, respectively, at right angles intersecting at *E*. On *C*, with *AE* as the radius, intersect the axis *AB* at *F* and *G*, the *foci*. Insert pins through the axis at *F* and *G*, and loop a thread or cord on them equal in length to the axis *AB*, so that when stretched, it reaches extremity *C* of the *conjugate axis*, as shown in dotted lines. Place a pencil inside the cord, as at *H*, and, by guiding the pencil in this manner, describe the ellipse.

Second Method

Along the edge of a piece of paper, mark off a distance *ac* equal to *AC*, one-half the major axis, and from the same point a distance *ab* equal to *CD*, one-half the minor axis, as shown in Figure 5-52. Place the paper to bring point *b* on the line *AB*, or major axis, and point *c* on the line *DE*, or minor axis. Lay off the position of point *a*.

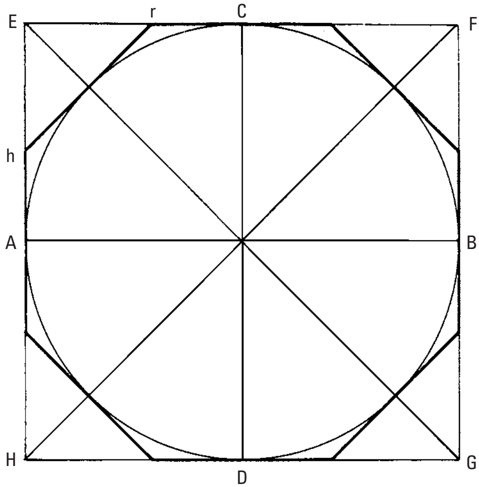


Figure 5-50 To circumscribe an octagon about a circle.

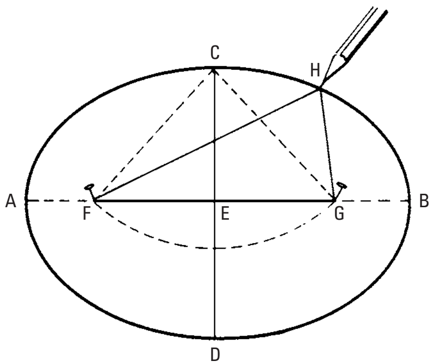


Figure 5-51 To describe an ellipse when the two axes are given.

By shifting the paper so that point *b* travels on the major axis and point *c* travels on the minor axis, any number of points in the curve may be found through which the curve may be traced.

Mensuration

As mentioned earlier, mensuration is the act, art, or process of measuring. It is the branch of mathematics that deals with finding the length of lines, the area of surfaces, and the volume of solids. Therefore, the problems that follow will be divided into three groups as follows:

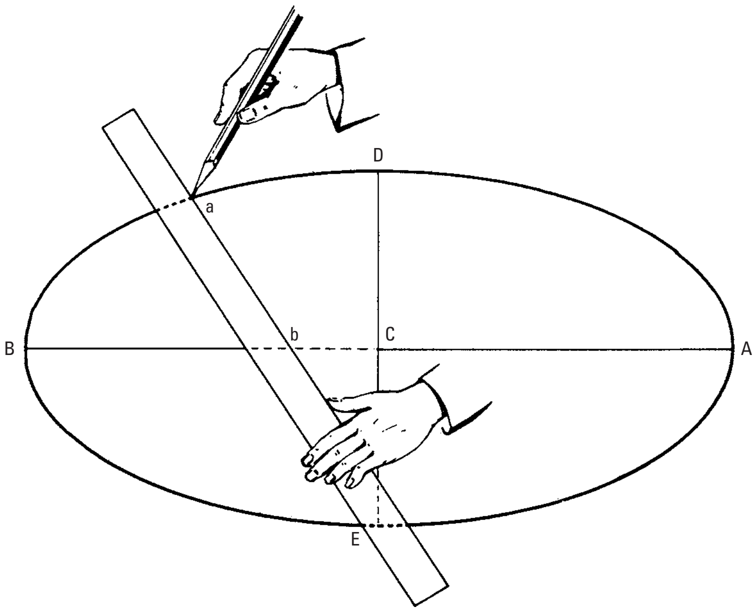


Figure 5-52 To describe an ellipse given the two axes, second method.

- Measurement of *lines*, one dimension (length)
- Measurement of *surfaces* (areas), two dimensions (length and width)
- Measurement of *solids* (volumes), three dimensions (length, width, and thickness)

Measurement of Lines—Length

Problem 1

To find the length of any side of a right triangle given the other two sides.

Rule: The length of the hypotenuse equals the square root of the sum of the squares of the two legs. The length of either leg equals the square root of the difference of the square of the hypotenuse and the square of the other leg.

Example The two legs of a right triangle measure 3 feet and 4 feet. Find the length of the hypotenuse. If the lengths of the hypotenuse and one leg are 5 feet and 4 feet, respectively, what is the length of the other leg?

In Figure 5-53A,

$$AB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ feet}$$

In Figure 5-53B,

$$BC = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4 \text{ feet}$$

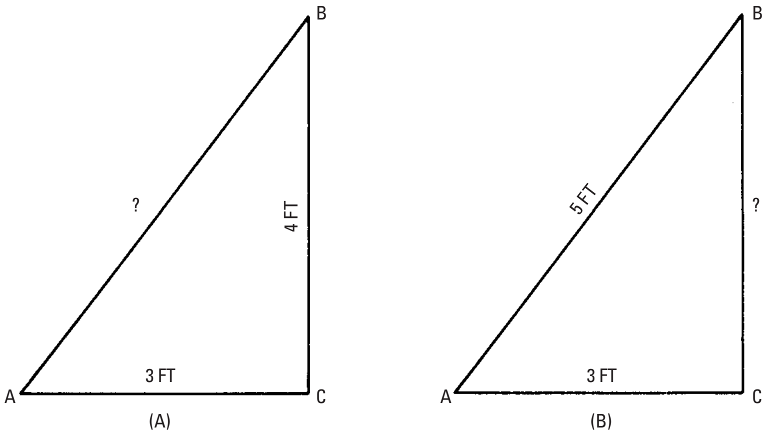


Figure 5-53 To find the length of any side of a right triangle given the other two sides.

Problem 2

To find the length of the circumference of a circle.

Rule: Multiply the diameter by 3.1416.

Example What length of molding strip is required for a circular window that is 5 feet in diameter?

$$5 \times 3.1416 = 15.7 \text{ feet}$$

Since the carpenter does not ordinarily measure feet in tenths, .7 should be reduced to inches. It corresponds to $8^1/_2$ inches from Table 5-23. That is, the length of molding required is 15 feet $8^1/_2$ inches.

Problem 3

To find the length of the arc of a circle.

Rule: Arc = .017453 × radius × central angle.

Example If the radius of a circle is 2 feet, what is the length of a 60° arc?

Table 5-23 Decimals of a Foot and Inches

Inch	*0 inch*	*1 inch*	*2 inches*	*3 inches*	*4 inches*	*5 inches*	*6 inches*	*7 inches*	*8 inches*	*9 inches*	*10 inches*	*11 inches*
0	0.0000	0.0833	0.1677	0.2500	0.3333	0.4167	0.5000	0.5833	0.6667	0.7500	0.8333	0.9167
1–16	0.0052	0.0885	0.1719	0.2552	0.3385	0.4219	0.5052	0.5885	0.6719	0.7552	0.8385	0.9219
1–8	0.0104	0.0937	0.1771	0.2604	0.3437	0.4271	0.5104	0.5937	0.6771	0.7604	0.8437	0.9271
3–16	0.0156	0.0990	0.1823	0.2656	0.3490	0.4323	0.5156	0.5990	0.6823	0.7656	0.8490	0.9323
1–4	0.0208	0.1042	0.1875	0.2708	0.3542	0.4375	0.5208	0.6042	0.6875	0.7708	0.8542	0.9375
5–16	0.0260	0.1094	0.1927	0.2760	0.3594	0.4427	0.5260	0.6094	0.6927	0.7760	0.8594	0.9427
3–8	0.0312	0.1146	0.1979	0.2812	0.3646	0.4479	0.5312	0.6146	0.6979	0.7812	0.8646	0.9479
7–16	0.0365	0.1198	0.2031	0.2865	0.3698	0.4531	0.5365	0.6198	0.7031	0.7865	0.8698	0.9531
1–2	0.0417	0.1250	0.2083	0.2917	0.3750	0.4583	0.5417	0.6250	0.7083	0.7917	0.8750	0.9583
9–16	0.0469	0.1302	0.2135	0.2969	0.3802	0.4635	0.5469	0.6302	0.7135	0.7969	0.8802	0.9635
5–8	0.0521	0.1354	0.2188	0.3021	0.3854	0.4688	0.5521	0.6354	0.7188	0.8021	0.8854	0.9688
11–16	0.0573	0.1406	0.2240	0.3073	0.3906	0.4740	0.5573	0.6406	0.7240	0.8073	0.8906	0.9740
3–4	0.0625	0.1458	0.2292	0.3125	0.3958	0.4792	0.5625	0.6458	0.7292	0.8125	0.8958	0.9792
13–16	0.0677	0.1510	0.2344	0.3177	0.4010	0.4844	0.5677	0.6510	0.7344	0.8177	0.9010	0.9844
7–8	0.0729	0.1562	0.2396	0.3229	0.4062	0.4896	0.5729	0.6562	0.7396	0.8229	0.9062	0.9896
15–16	0.0781	0.1615	0.2448	0.3281	0.4115	0.4948	0.5781	0.6615	0.7448	0.8281	0.9115	0.9948

Solution:

$$2 \times .017453 \times 60 = 2.094,$$

or approximately 2 feet $1^1/_8$ inches

Problem 4

To find the rise of an arc.

Rule: Rise of an arc =

$$\sqrt{(4 \times \text{radius}^2) - \text{length}}$$

Example If the radius of a circle is 2 feet, what is the rise at the center of a 2-foot chord?

Solution:

$$^1/_2\sqrt{(4 \times 2^2) - 2} = {}^1/_2\sqrt{14} = 1.87 \text{ feet}$$
$$= 1 \text{ feet } 10^1/_2 \text{ inches}$$

Measurement of Surfaces—Area

Problem 5

To find the area of a square.

Rule: Multiply the base by the height.

Example What is the area of a square whose side is 5 feet (see Figure 5-54)?

$$5 \times 5 = 25 \text{ square feet}$$

Problem 6

To find the area of a rectangle.

Rule: Multiply the base by the height (that is, width by length).

Example What is the floor area of a porch 5 feet wide and 12 feet long (see Figure 5-55)?

$$5 \times 12 = 60 \text{ square feet}$$

Problem 7

To find the area of a parallelogram.

Rule: Multiply the base by the perpendicular height.

Example What is the area of a 5-foot × 12-foot parallelogram (see Figure 5-56)?

$$5 \times 12 = 60 \text{ square feet}$$

Problem 8

To find the area of a triangle (see Figure 5-57)

Rule: Multiply the base by one-half the altitude.

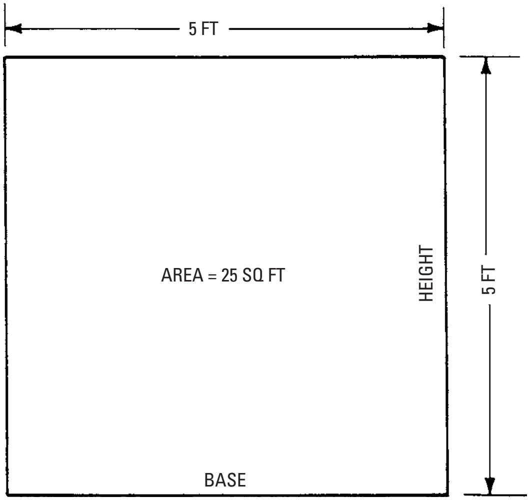


Figure 5-54 To find the area of a square.

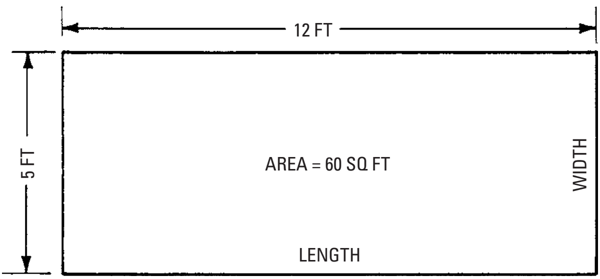


Figure 5-55 To find the area of a rectangle.

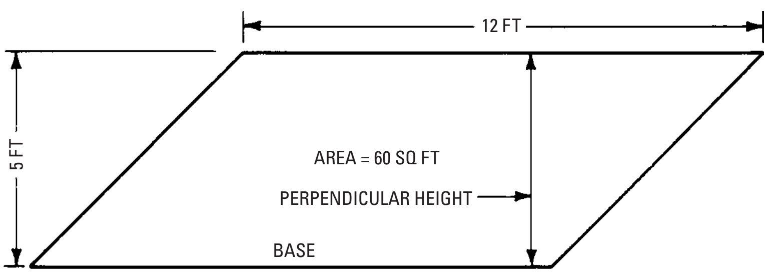


Figure 5-56 To find the area of a parallelogram.

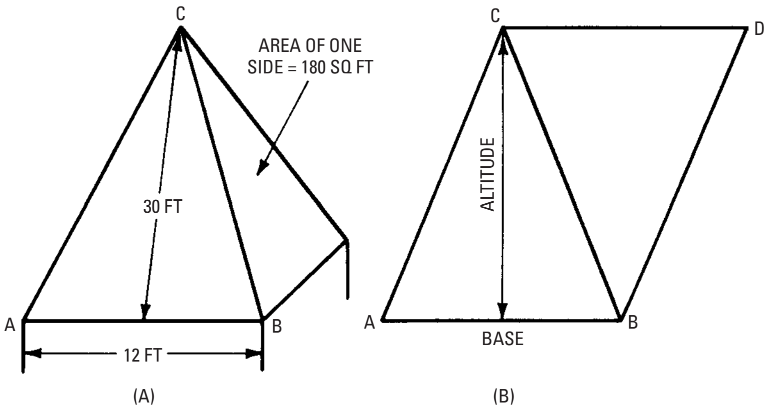


Figure 5-57 To find the area of a triangle (equal to ½ area of parallelogram ABDC).

Example How many square feet of sheathing are required to cover a church steeple having four triangular sides?

Problem 9

To find the area of a trapezoid.

Rule: Multiply one-half the sum of the two parallel sides by the perpendicular distance between them.

Example What is the area of the trapezoid shown in Figure 5-58?

LA and *FR* are the parallel sides, and *MS* is the perpendicular distance between them. Therefore,

$$\text{area} = {}^1/_2\,(LA + FR) \times MS$$

$$\text{area} = {}^1/_2\,(8 + 12) \times 6 = 60 \text{ square feet}$$

Problem 10

To find the area of a trapezium.

Rule: Draw a diagonal, dividing the figure into triangles. Measure the diagonal and the altitudes, and find the area of the triangles. The sum of these areas is then the area of the trapezium.

Example What is the area of the trapezium shown in Figure 5-59? (Draw diagonal *LR* and altitudes *AM* and *FS*.)

$$\text{area of triangle } ALR = {}^1/_2\,(12 \times 9) = 54 \text{ square feet}$$

$$\text{area of triangle } LRF = {}^1/_2\,(12 \times 6) = 36 \text{ square feet}$$

$$\text{area of trapezium } LARF = ALR + LRF = 36 + 54$$

$$= 90 \text{ square feet}$$

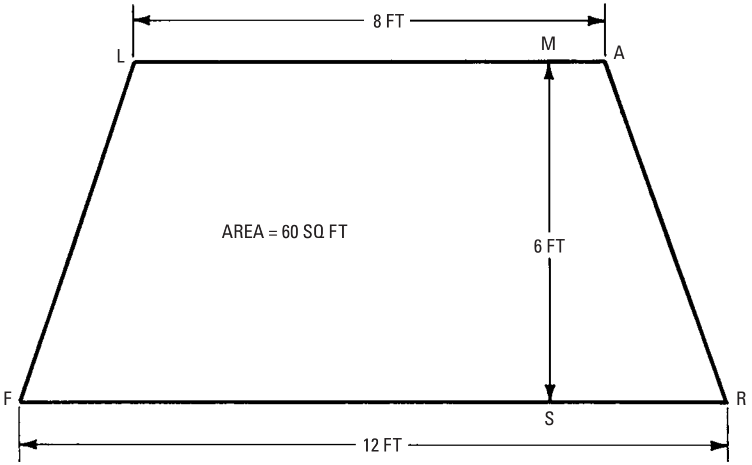


Figure 5-58 To find the area of a trapezoid.

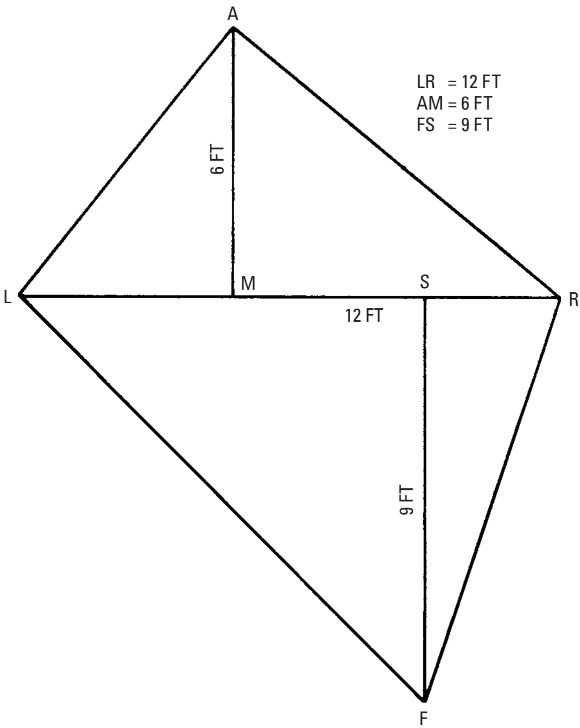


Figure 5-59 To find the area of a trapezium.

Problem 11

To find the area of any irregular polygon.

Rule: Draw diagonals, dividing the figure into triangles, and find the sum of the areas of these triangles.

Problem 12

To find the area of any regular polygon, such as shown in Figure 5-60, when the length of only one side is given

Rule: Multiply the square of the sides by the figure for "area when side = 1" opposite the particular polygon in Table 5-24.

Example What is the area of an octagon (8-sided polygon) whose sides are 4 feet in length?

In Table 5-24 under 8 find 4.828. Multiply this by the square of one side.

$$4.828 \times 4^2 = 77.25 \text{ square feet}$$

Problem 13

To find the area of a circle (see Figure 5-61).

Rule: Multiply the square of the diameter by 0.7854.

Example How many square feet of floor surface are there in a 10-foot circular floor?

$$10^2 \times 0.7854 = 78.54 \text{ square feet}$$

Problem 14

To find the area of a sector of a circle.

Rule: Multiply the arc of the sector by one-half the radius.

Example How much tin is required to cover a 60° section of a 10-foot circular deck?

$$\text{length of } 60° \text{ arc} = {}^{60}/_{30} \text{ of } 3.1416 \times 10 = 5.24 \text{ feet}$$

$$\text{tin required for } 60° \text{ sector} = 5.24 \times {}^{1}/_{2} \times 5$$
$$= 13.1 \text{ square feet}$$

Problem 15

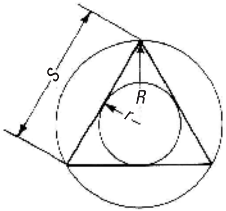
To find the area of a segment of a circle.

Rule: Find the area of the sector that has the same arc, and also find the area of the triangle formed by the radii and chord. Take the sum of these areas if the segment is greater than 180°. Take the difference if the segment is less than 180°.

Problem 16

To find the area of a ring.

Rule: Take the difference between the areas of the two circles.

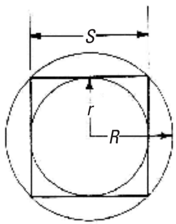


Steel square cut to miter sides at 30°

$R = 2r$, or $.577\ S$

$\text{AREA} = .433\ S^2$
$= 1.299\ R^2$
$= 5.196\ r^2$
$= \frac{(R+r)}{2}\ S$

(A) Equilateral triangle (3 sides).

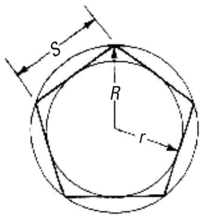


Steel square cut to miter sides at 45°

$R = .707\ S$, or $1.414\ r$

$\text{AREA} = S^2$
$= 2R^2$
$= 4r^2$

(B) Square (4 sides).



Steel square cut to miter sides at 36°

$R = .851\ S$, or $1.236\ r$

$\text{AREA} = 1.72\ S^2$
$= 2.378\ R^2$
$= 3.633\ r^2$

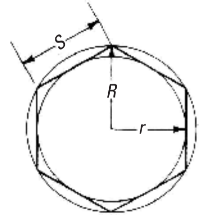
(C) Pentagon (5 sides).

Steel square cut to miter sides at 30°

$R = S$, or $1.155\ r$

$\text{AREA} = 2.598\ S^2$
$= 2.598\ R^2$
$= 3.464\ r^2$

(D) Hexagon (6 sides).

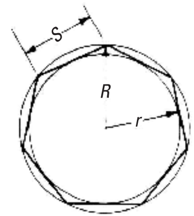


Steel square cut to miter corners at 25° 43'

$R = 1.152\ S$, or $1.11\ r$

$\text{AREA} = 3.634\ S^2$
$= 2.736\ R^2$
$= 3.371\ r^2$

(E) Heptagon (7 sides).



Steel square cut to miter corners at 22° 30'

$R = 1.307\ S$, or $1.082\ r$

$\text{AREA} = 4.828\ S^2$
$= 2.828\ R^2$
$= 3.314\ r^2$

(F) Octagon (8 sides).

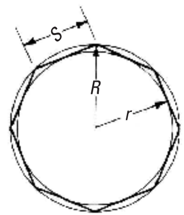


Figure 5-60 Regular polygons.

Table 5-24 Regular Polygons

Number of sides	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***
Area when side = 1	0.433	1.0	1.721	2.598	3.634	4.828	6.181	7.694	9.366	11.196

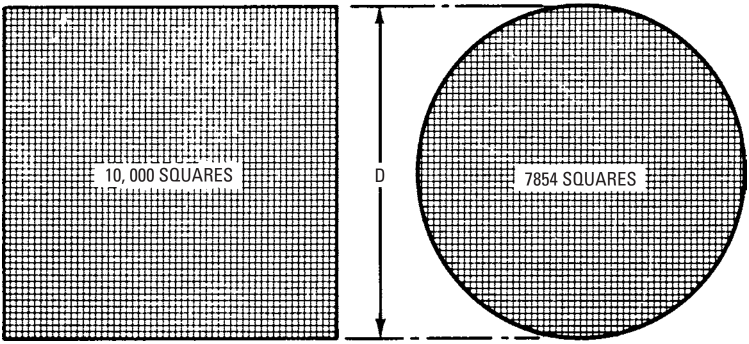


Figure 5-61 The decimal 0.7854 is used to find the area of a circle. If a square is divided into 10,000 equal parts (small squares), then a circle with a diameter D equal to one side of the large square will contain 7854 small squares. Therefore, if the area of the large square is 1 square inch, then the area of the circle will be $^{7854}/_{10,000}$, or 0.7854 square inch.

Problem 17

To find the area of an ellipse.

Rule: Multiply the product of the two diameters by 0.7854.

Example What is the area of an ellipse whose two diameters are 10 inches and 6 inches?

$$10 \times 6 \times 0.7854 = 47.12 \text{ square inches}$$

Problem 18

To find the circular area of a cylinder.

Rule: Multiple 3.1416 by the diameter and by the height.

Example How many square feet of lumber are required for the sides of a cylindrical tank (see Figure 5-62) that is 8 feet in diameter and 12 feet high? How many 4-inch × 12-feet pieces will be required?

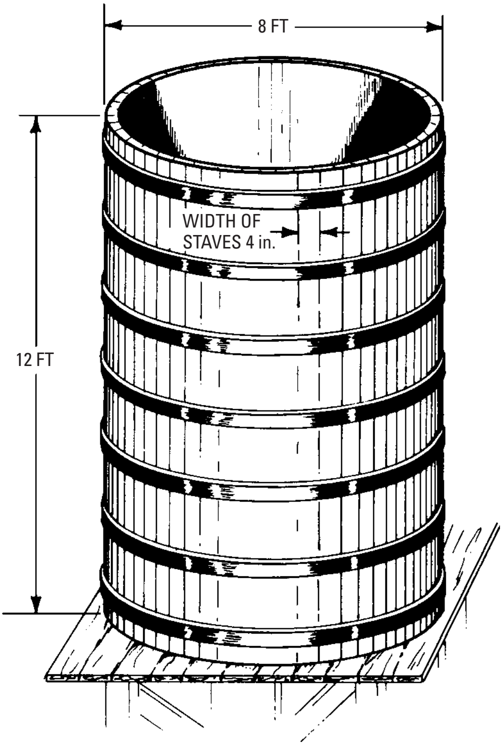


Figure 5-62 To find the area of a cylinder.

$$\text{cylindrical surface} = 3.1416 \times 8 \times 12 = 302 \text{ square feet}$$

$$\text{circumference of tank} = 3.1416 \times 8 = 25.1 \text{ feet}$$

$$\text{number of } 4'' \times 12' \text{ pieces} = \frac{25.1 \times 12}{4} = 25.1 \times 3 = 75.3$$

Problem 19

To find the area of a cone (see Figure 5-63).

Rule: Multiply 3.1416 by the diameter of the base and by one-half the slant height.

Example A conical spire with a base 10 feet in diameter and an altitude of 20 feet is to be covered. Find the area of the surface to be covered.

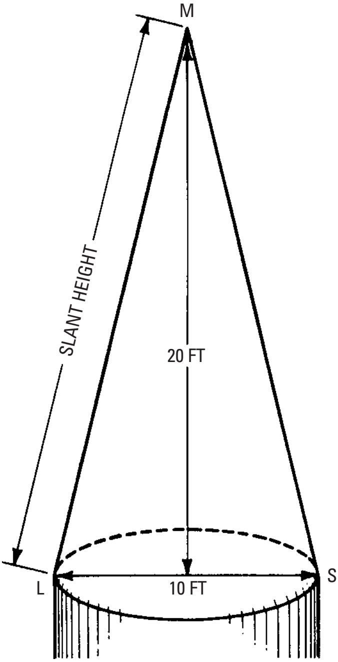


Figure 5-63 To find the surface area of a cone.

$$\text{slant height} = \sqrt{5^2 + 20^2} = \sqrt{425} = 20.62 \text{ feet}$$

$$\text{circumference of base} = 3.1416 \times 10 = 31.416 \text{ feet}$$

$$\begin{aligned}\text{area of conical surface} &= 31.416 \times {}^1/_2 \times 20.62\\ &= 324 \text{ square feet}\end{aligned}$$

Problem 20

To find the area of the frustum of a cone (see Figure 5-64)

Rule: Multiply one-half the slant height by the sum of the circumference.

Example A tank is 12 feet in diameter at the base, 10 feet at the top, and 8 feet high. What is the area of the slant surface?

$$\begin{aligned}\text{circumference of 10-foot diameter} &= 3.1416 \times 10\\ &= 31.416 \text{ feet}\end{aligned}$$

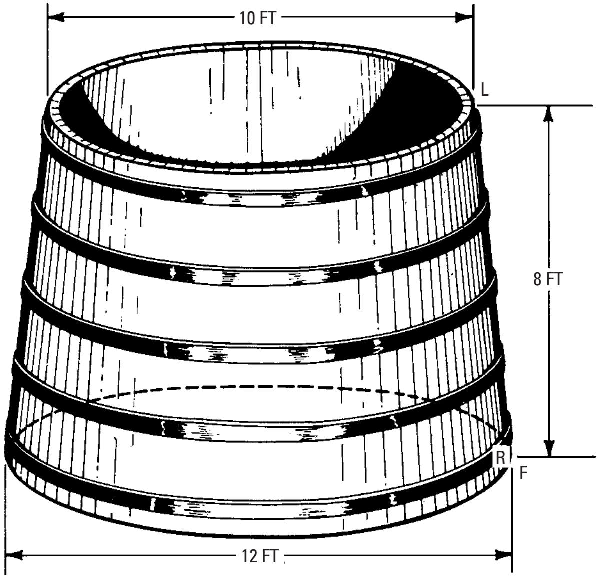


Figure 5-64 To find the area of the frustum of a cone.

$$\text{circumference of 12-foot diameter} = 3.1416 \times 12$$
$$= 37.7 \text{ feet}$$

$$\text{sum of circumferences} = 69.1 \text{ feet}$$

$$\text{slant height} = \sqrt{1^2 + 8^2} = \sqrt{65} = 8.12$$

$$\text{slant surface} = \text{sum of circumferences} \times {}^1/_2 \text{ slant height}$$

$$\text{slant surface} = 69.1 \times {}^1/_2 \times 8.12 = 280 \text{ square feet}$$

Measurement of Solids—Volume

Problem 21

To find the volume of a rectangular solid.

Rule: Multiply the length, width, and thickness together.

Example What is the volume of a 4-inch × 8-inch × 12-foot timber? (Before applying the rule, reduce all dimensions to feet.)

$$4 \text{ inches} = {}^1/_3 \text{ foot}$$

$$8 \text{ inches} = {}^2/_3 \text{ foot}$$

$$\text{volume of timber} = {}^1/_3 \times {}^2/_3 \times 12 = 2.67 \text{ cubic feet}$$

If the timber were a piece of oak weighing 48 pounds per cubic foot, the total weight would be calculated as follows:

$48 \times 2.67 = 128$ pounds

Problem 22

To find the volume of a rectangular wedge.

Rule: Find the area of one of the triangular ends, and multiply the area by the distance between the ends.

Example An attic has the shape of a rectangular wedge. What volume storage capacity would there be for the proportions shown in Figure 5-65? In the illustration, the boundary of the attic is *LARFMS*.

Area of triangular end $MLA = 20 \times {}^{10}/_{2} = 100$ square feet

Volume of attic $= 100 \times 40 = 4000$ cubic feet.

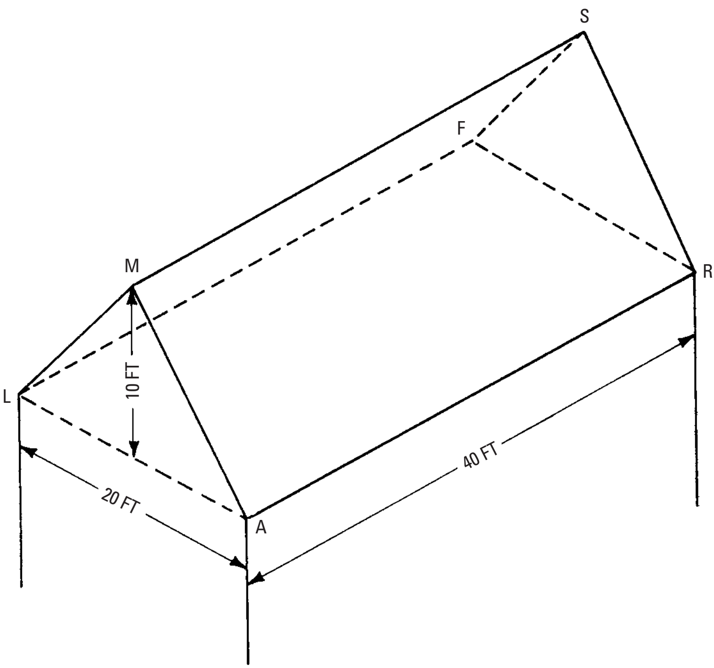


Figure 5-65 To find the volume of a rectangular wedge.