

KEYWORDS

Equivalent

Unitary

Ratio

Ratio

Ratio is the comparison of two or more quantities expressed in the same units.

The ratio of a to b is written as $a:b$ (or sometimes $\frac{a}{b}$).

Order is important so that $a:b$ is different to $b:a$. The value of the ratio $a:b$ is $\frac{a}{b}$, of $b:a$ is $\frac{b}{a}$.

Examples of ratios are 2:5, 5:2, 3:4, 4:3, 7:2. The value of the ratio 2:5 is $\frac{2}{5}$, while the value of 5:2 is $\frac{5}{2}$.

Equivalent Ratios

Ratios are simplified in the same way as fractions—we divide (or multiply) both components of the ratio by the same number (except 0).



For Example

Simplify:

1 6:4

2 4:6

3 30:50

4 16:64

5 120:10

6 40:20:10

7 $\frac{3}{4}:\frac{1}{4}$

8 $\frac{2}{3}:1$

9 $1\frac{1}{2}:\frac{1}{4}$

10 $1\frac{1}{2}:2$

11 $\frac{4}{5}:1\frac{1}{2}$

12 0.2:0.8

13 0.5:1.5:4

1 $6:4 = \frac{6}{2}:\frac{4}{2}$ (dividing both by 2)
 $= 3:2$

2 $4:6 = \frac{4}{2}:\frac{6}{2}$
 $= 2:3$

3 $30:50 = 3:5$ (divide both parts by 10)

4 $16:64 = 1:4$ (divide by 16)

5 $120:10 = 12:1$
(cross off zeros—divide by 10)

6 $40:20:10 = 4:2:1$ (divide all by 10)

7 $\frac{3}{4}:\frac{1}{4} = \frac{3}{4} \times 4:\frac{1}{4} \times 4$ (multiplying both by 4)
 $= 3:1$

8 $\frac{2}{3}:1 = \frac{2}{3} \times 3:1 \times 3$
 $= 2:3$

9 $\frac{1}{2}:\frac{1}{4} = \frac{1}{2} \times 4:\frac{1}{4} \times 4$
 $= 2:1$

or $\frac{1}{2}:\frac{1}{4} = \frac{2}{4}:\frac{1}{4}$
 $= 2:1$ (multiplying both by 4)

10 $1\frac{1}{2}:2 = 1\frac{1}{2} \times 2:2 \times 2$
 $= 3:4$

or $1\frac{1}{2}:2 = \frac{3}{2}:2$
 $= \frac{3}{2}:\frac{4}{2}$
 $= 3:4$ (multiplying by 2)

11 $\frac{4}{5}:1\frac{1}{2} = \frac{4}{5} \times 10:1\frac{1}{2} \times 10$
 $= \frac{8}{10} \times 10:\frac{15}{10} \times 10$
 $= 8:15$

Multiply both parts by 10 as this is the LCD of 5 and 2.

12 $0.2:0.8 = 0.2 \times 10:0.8 \times 10$
 $= 2:8$
 $= 1:4$ (dividing by 2)

13 $0.5:1.5:4 = 5:15:40$ (multiplying by 10)
 $= 1:3:8$ (dividing by 5)

Sometimes we find the value of the pronumeral to complete the simplification.



For Example

Find the value of pronumerals:

1 $x:4 = 15:20$ 2 $4:28 = 2:x$

1 $15:20 = 3:4$ 2 $4:28 = 2:14$
 $\therefore x = 3$ $\therefore x = 14$

Often we have to change one part of the ratio so that both parts have the same units. We then simplify the resulting ratio.



For Example

Simplify:

- 1 20 cents:80 cents
- 2 40 cents:\$2
- 3 \$1.20:\$6
- 4 6 days:3 weeks
- 5 4 cm:1 m
- 6 2 L:150 mL
- 7 8 secs:2 min
- 8 4 hours:1 day
- 9 $1\frac{1}{2}$ mL:1 L
- 10 400 kg:2 t
- 11 \$20:\$40:\$80
- 12 7.2 L:21.6 L
- 13 $3x:12x$
- 14 $21ab:14ac$

15 $\frac{1}{p}:p$

16 $16ab:4ab^2$

1 20 cents:80 cents = 20:80
 = 1:4 (divide by 20)

2 40 cents:\$2 = 40 cents:200 cents
 = 40:200
 = 1:5 (divide by 40)

3 \$1.20:\$6 = 1.2:6
 = 6:30 (multiply by 5)

[or 120c:600c = 1:5]
 = 1:5 (divide by 6)

4 6 days:3 weeks = 6 days:21 days
 = 6:21
 = 2:7

5 4 cm:1 m = 4 cm:100 cm
 = 4:100
 = 1:25

6 2 L:150 mL = 2000 mL:150 mL
 = 2000:150
 = 200:15
 = 40:3 (dividing by 5)

7 8 secs:2 min = 8 secs:120 secs
 = 8:120
 = 1:15

8 4 hours:1 day = 4 hours:24 hours
 = 4:24
 = 1:6

9 $1\frac{1}{2}$ mL:1 L = $1\frac{1}{2}$ mL:1000 mL
 = $1\frac{1}{2}$:1000
 = 3:2000 (multiply by 2)

10 400 kg:2 t = 400 kg:2000 kg
 = 400:2000
 = 4:20
 = 1:5

t is the symbol for metric tonne = 1000 kg

$$\begin{aligned} 11 \quad \$20:\$40:\$80 &= 20:40:80 \\ &= 1:2:4 \end{aligned}$$

$$\begin{aligned} 12 \quad 7.2 \text{ L}:21.6 \text{ L} &= 7.2:21.6 \\ &= 72:216 \text{ (multiply by 10)} \\ &= 1:3 \text{ (divide by 72)} \end{aligned}$$

$$\begin{aligned} 13 \quad 3x:12x &= \frac{3x}{x}:\frac{12x}{x} \text{ (divide both parts by } x\text{)} \\ &= 3:12 \\ &= 1:4 \end{aligned}$$

$$\begin{aligned} 14 \quad 21ab:14ac &= \frac{21ab}{a}:\frac{14ac}{a} \text{ (divide by } a\text{)} \\ &= 21b:14c \\ &= 3b:2c \text{ (divide by 7)} \end{aligned}$$

$$\begin{aligned} 15 \quad \frac{1}{p}:p &= \frac{1}{p} \times p:p \times p \\ &= 1:p^2 \end{aligned}$$

$$\begin{aligned} 16 \quad 16ab:4ab^2 &= \frac{\overset{4}{\cancel{16}}\overset{1}{\cancel{a}}\overset{1}{\cancel{b}}}{\underset{1}{\cancel{a}}\underset{1}{\cancel{b}}}\frac{\overset{1}{\cancel{16}}\overset{1}{\cancel{a}}\overset{1}{\cancel{b}}\overset{1}{\cancel{b}}}{\underset{1}{\cancel{a}}\underset{1}{\cancel{b}}\underset{1}{\cancel{b}}} \\ &\text{(divide by } 4ab\text{)} \\ &= 4:b \end{aligned}$$

Finding a Ratio of Two or More Quantities

From the information in a problem, a ratio is formed and simplified if necessary.



For Example

- 1 A netball team won 8 games in a season and lost 4 games. What is the ratio of games won to games lost?
- 2 Sylvio mixes 6 buckets of gravel, 4 buckets of sand and 2 buckets of cement into a batch of concrete. Determine the ratio of gravel to sand to cement that was used.

- 3 A street in a new subdivision has 40 blocks of land, of which 25 have been built on. What is the ratio of occupied to vacant blocks?
- 4 The results of an election were announced:

Candidate	No. of votes
Christine Howard	16 000
Noel Horton	12 000
Peter Chapman	20 000

What is the ratio of the votes for:

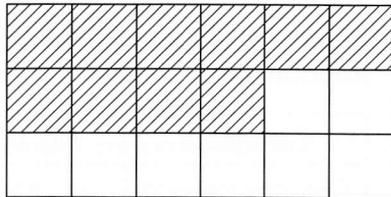
- a Christine Howard to Noel Horton?
 - b Peter Chapman to the total votes?
- 5 In a class survey the hair colour of boys and girls was recorded:

Colour	Boys	Girls
Blonde	3	4
Brown	5	7
Black	2	4
Red	2	1

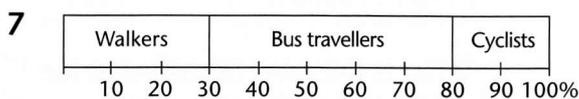
- a Write down the number of students:
 - i who are boys
 - ii who are girls
 - iii in the class.
- b What is the ratio of:
 - i Blonde boys to blonde girls?
 - ii Blonde girls to black-haired girls?
 - iii Brown-haired to black-haired to red-haired students?
 - iv Non-blond students to the total number of students?

6 Find the ratio of:

a Shaded squares to unshaded squares:



b Shaded squares to total number of squares.

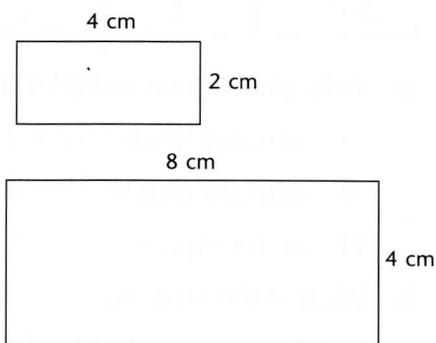


Students in Year 7 were surveyed to determine their method of travelling to school each day and the results recorded in the above graph. Find the ratio of:

a Bus travellers to walkers

b Bus travellers to the total number of students surveyed.

8



What is the ratio of the:

a Lengths of the two rectangles?

b Perimeters of the two rectangles?

c Areas of the two rectangles?

1 $8:4 = 2:1$

2 $6:4:2 = 3:2:1$

3 Vacant = $40 - 25$
= 15

$\therefore 25:15 = 5:3$

4 a $16\,000:12\,000 = 16:12$
= 4:3

b Total = $16\,000 + 12\,000 + 20\,000$
= 48 000

\therefore Ratio = $20\,000:48\,000$
= 20:48
= 5:12

5 a i Boys = $3 + 5 + 2 + 2$
= 12

\therefore There were 12 boys.

ii Girls = $4 + 7 + 4 + 1$
= 16

\therefore There were 16 girls.

iii Total = $12 + 16$
= 28

\therefore There were 28 students.

b i 3:4

ii $4:4 = 1:1$

iii $12:6:3 = 4:2:1$

iv $(28 - 7):28 = 21:28$
= 3:4

6 a Shaded = 10, unshaded = 8
 $\therefore 10:8 = 5:4$

b Shaded = 10, total = 18
 $\therefore 10:18 = 5:9$

7 a Bus travellers = 50, walkers = 30
Ratio = $50\%:30\%$
= 50:30
= 5:3

b Bus travellers = 50, total students = 100
Ratio = $50\%:100\%$
= 50:100
= 1:2

8 a $4 \text{ cm}:8 \text{ cm} = 4:8 = 1:2$

b Perimeter of small rectangle $= 2(4 + 2)$
 $= 2(6) = 12$

\therefore The perimeter is 12 cm.

Perimeter of large rectangle $= 2(8 + 4)$
 $= 2(12)$
 $= 24$

\therefore The perimeter is 24 cm.

\therefore Ratio of perimeters $= 12 \text{ cm}:24 \text{ cm}$
 $= 12:24$
 $= 1:2$

c Area of small rectangle $= 4 \times 2$
 $= 8$

\therefore The area is 8 cm^2 .

Area of large rectangle $= 8 \times 4$
 $= 32$

\therefore The area is 32 cm^2 .

\therefore Ratio of areas $= 8 \text{ cm}^2:32 \text{ cm}^2$
 $= 8:32$
 $= 1:4$

Using Ratios to Solve Problems

The Unitary Method

The **unitary method** can be used in ratio problems. It involves finding the value of one item (or one *unit*) and then multiplying this value to find the required answer.

For Example

- 1 The ratio of cats to dogs is 4:5. If there are 30 dogs, how many cats?

- 2 On a bus trip the ratio of males to females is 4:7. If there were 28 females on the bus, how many males?

- 3 Three sums of money are in the ratio of 2:3:5. If the smallest amount is \$4.80, find the largest amount.

- 4 It is known that a metal alloy is made of copper, tin and zinc in the ratio 14:17:10. In a certain batch, 85 kg of tin is used. How much copper and zinc will be required to make the alloy?

- 5 Joe's will stated that his estate should be divided such that for each \$2 that Carol receives, Keith should receive \$5. If Keith gained \$74 000 from Joe's estate, how much will Carol receive?

- 6 The ratio of the ages of three brothers is 10:8:5. If the youngest is 15 years old, how old is the eldest?

1 Cats to dogs $= 4:5$ [Order is important.]

Now, 30 dogs:

\therefore 5 parts $= 30$

1 part $= 6$

4 parts $= 4 \times 6$
 $= 24$

\therefore There are 24 cats. [Check: $24:30 = 4:5$]

2 Males to females $= 4:7$

Now, 28 females:

\therefore 7 parts $= 28$

1 part $= 4$

4 parts $= 16$ [Check: $16:28 = 4:7$]

\therefore There are 16 males.

3 Sums of money $= 2:3:5$

\therefore smallest amount $= 2$ parts

\therefore 2 parts $= \$4.80$

1 part $= \$2.40$

\therefore 5 parts $= \$12$

\therefore \$12 is largest amount.

- 4 Copper, tin, zinc = 14:17:10
 \therefore 17 parts = 85
 1 part = 5
 \therefore 14 parts = 70
 and 10 parts = 50
 \therefore 70 kg copper and 50 kg zinc are required.
- 5 Carol:Keith = \$2:\$5
 = 2:5
- \therefore Keith's share is 5 parts
 (Carol's share is 2 parts)
- \therefore 5 parts = \$74 000
 1 part = \$14 800 ($74\,000 \div 5$)
 2 parts = \$29 600 ($14\,800 \times 2$)
- \therefore Carol's share of the estate is \$29 600.
- 6 Brothers' ages = 10:8:5 (5 is the youngest)
 \therefore 5 parts = 15
 1 part = 3
 10 parts = 30
 \therefore The eldest brother is 30 years old.

Dividing Quantities in a Given Ratio

To divide quantities in a given ratio, find the total of the parts and then use the unitary method.

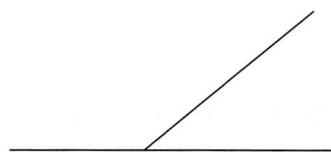


For Example

- 1 Divide \$250 in the ratio of 7:3.
- 2 Share \$72 in the ratio of 4:3:2.
- 3 If \$200 is to be split in the ratio of 3:2, find the smaller amount?
- 4 A 2 L container is used to make up an orange fruit juice drink. 500 mL of concentrate is poured into the container, and then water is used to fill the remainder of the container:

- a What is the ratio of concentrate to water in the fruit juice drink?
- b Two-hundred and forty millilitres of the fruit juice drink is poured into a glass. How much of the drink is water?

5



The ratio of two supplementary angles is 4:5. What is the size of each angle?

- 6 Two sisters, Margaret and Robyn, contribute \$8000 and \$4000 respectively to buy a car. Three years later they decide to sell the car for \$7200 and split the money in the same ratio as their investment. What was Margaret's share?
- 7 The ratio of a father's age to that of his son is 9:2. If the sum of their ages is 44, how old is the father?
- 8 In an orchard the ratio of orange trees to lemon trees is 3:1, while the ratio of lemon trees to mandarin trees is 2:5:
 - a What is the ratio of orange trees to lemon trees to mandarin trees?
 - b If there are 390 citrus trees, how many of each variety are in the orchard?

- 1 Total parts = $7 + 3$
 $= 10$
 \therefore 10 parts = \$250
 $\therefore \frac{7}{10} \times \$250 = \$175$
 $\frac{3}{10} \times \$250 = \75
 \therefore The two amounts are \$175 and \$75.

2 Total parts = $4 + 3 + 2$
 $= 9$

$\therefore 9 \text{ parts} = \72

$\therefore \frac{4}{9} \times \$72 = \$32$

$\frac{3}{9} \times \$72 = \24

$\frac{2}{9} \times \$72 = \16

$\therefore \$32, \$24, \$16$

3 5 parts = \$200

$\therefore \frac{2}{5} \times \$200 = \$80$

\therefore The smaller amount is \$80.

4 a Water = $2000 - 500$
 $= 1500$

$\therefore 1500 \text{ mL water}$

\therefore Concentrate:water = $500 \text{ mL}:1500 \text{ mL}$
 $= 1:3$

\therefore The ratio of concentrate to water is 1:3.

b Total parts = $1 + 3$
 $= 4$

$\therefore 4 \text{ parts} = 240 \text{ mL}$

$\therefore \frac{3}{4} \times 240 = 180$

$\therefore 180 \text{ mL of the drink is water.}$

5 Supplementary angles add to 180° .

As $4 + 5 = 9$, there are 9 parts:

$\therefore 9 \text{ parts} = 180^\circ$

$\frac{4}{9} \times 180 = 80$

$\frac{5}{9} \times 180 = 100$

\therefore The angles are 80° and 100° .

6 Ratio of investments = $\$8000:\4000
 $= 2:1$

\therefore Total parts = $2 + 1$

$= 3$

$\therefore \frac{2}{3} \times \$7200 = \$4800$

\therefore Margaret's share of the car is worth \$4800.

7 Total parts = $9 + 2$
 $= 11$

$\therefore \frac{9}{11} \times 44 = 36$

\therefore The father is 36 years old.

8 a Orange:lemon = $3:1 = 6:2$,
 lemon:mandarin = $2:5$

$\therefore 6:2:5$ is the ratio of orange to lemon to mandarin.

b Total parts = $6 + 2 + 5$
 $= 13$

$\therefore 13 \text{ parts} = 390$

$\frac{6}{13} \times 390 = 180$

$\frac{2}{13} \times 390 = 60$

$\frac{5}{13} \times 390 = 150$

\therefore There are 180 orange trees, 60 lemon trees and 150 mandarin trees in the orchard.