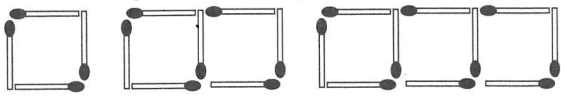
NUMBER AND ALGEBRA
Patterns and algebra

1 To complete a **pattern of numbers** we first determine the rule and then use it to find other numbers.
Examples:

a Complete the sequence 32, 51, 70, 89, ____
The sequence is counting forward by 19.
Next number = 89 + 19
= 89 + 20 – 1
= 109 – 1
= 108
The next number is 108.

b What is the missing number in this sequence: 256,128, 64, ___, 16, 8
The pattern is dividing by 2.
Missing number = 64 ÷ 2
= 32
The missing number is 32.

2 A **pattern of shapes** can be summarised into a table.
Example: Matches are used to form a pattern of squares.



a How many matches are needed to make 4 squares?
The matches used are 4, 7, 10, ?
The next number is 13

b Complete the table:

Squares	1	2	3	4	5	6	7
Matches	4	7	10	13			

The three entries are 16, 19, 22

c Write in words the rule for the bottom row.
The bottom number is 3 times the top number plus 1.

d How many matches are needed for 20 squares?
Using the rule in **c**, $3 \times 20 + 1 = 61$

3 A table can be completed by **determining the rule, or pattern.**
Example:

Top	1	2	3	4	5	6
Bottom	6	10	14	18	22	?

a What is the rule?
The bottom number is four times the top number plus 2.

b What is the missing number?
$4 \times 6 + 2 = 24 + 2$
$= 26$

4 Number sentences can be completed by **finding the missing value.**
Examples: Find the missing value.

a $5 + \square = 3 \times 4$
As $3 \times 4 = 12$, then the missing number is 7 as $5 + 7 = 12$

b $6 \times \square = 3$
Replace the missing number with the phrase, 'what number': 6 times 'what number' is 3. This means the number is $\frac{1}{2}$

5 An **unknown number** can be found.
Example: I am thinking of a number so that when I double it and add 5, I get 17.
What is the number?
2 times 'what number' + 5 = 17
This means, 2 times 'what number' = 12
The number is 6.

6 A solution can be checked by **substituting different numbers** in the original question.
Example: Half of a certain number plus six is equal to ten. Find the number.
A 2 B 4 C 6 D 8
Check each of the choices:
2: Half of $2 + 6 = 1 + 6 \neq 10$
3: Half of $4 + 6 = 2 + 6 \neq 10$
6: Half of $6 + 6 = 3 + 6 \neq 10$
8: Half of $8 + 6 = 4 + 6 = 10$
The number is 8.

7 **Inverse operations** are useful when solving number sentences. Inverse operations are addition and subtraction, multiplication and division.
Examples: Find the missing value in

a $95 + \square = 198$
The inverse of addition is subtraction:
$\square = 198 - 95 = 103$

b $\square \div 0.4 = 0.6$
The inverse of division is multiplication
$\square = 0.6 \times 0.4 = 0.24$

16 Fastest trip is the 0817 from Jeffersen arriving in Bradley at 0836.
This is a trip time of 36 – 17 = 19 min

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1 The pattern is counting forward by 13:
91 + 13 = 104
The missing value is 104.

2 The pattern is counting forward by $\frac{2}{5}$:

$$\frac{4}{5} + \frac{2}{5} = \frac{6}{5}$$
$$= 1\frac{1}{5}$$

The missing value is $1\frac{1}{5}$.

3 The pattern is counting forward by 0.7:
1.6 + 0.7 = 2.3
The missing value is 2.3.

4 Continue the pattern: 16, 12, 8, 4, 0, –4, ...
The sixth number is –4.

5 The rule: 'bottom number = double the top number and then add 7'
$= 2 \times 3 + 7$
$= 6 + 7$
$= 13$
The value of X is 13.

6 Pattern of matches: 3, 5, 7, 9, ___
The next number is 11
This means 11 matches for 5 triangles.

7 Continuing the pattern:, ... 9, 11, 13, 15, 17
The value of X is 17.
[Also, the rule is Matches = 2 × triangles + 1]

8 Top row is counting forward by 5, and the bottom row is counting backward by 5.
This means X = 27 and Y = 33.

9 The pattern is 1, 3, 6, 10, ...
The differences are 2, 3, 4, etc.
The next number is 10 + 5 = 15.

10 The pattern of dots is 1, 3, 6, 10, 15, 21, ...
The number X is 21.

11 As 295 – ___ = 86,
then 295 – 86 = ___
The missing number is 209.

$$\begin{array}{r} 2\,{}^{8}\not{9}\,{}^{1}5 \\ -\quad 8\,6 \\ \hline 2\,0\,9 \end{array}$$

12 As 4 × __ – 2 = 18 then 4 × __ = 20
then the missing number is 5.

13 Work backwards from Jack's answer by using the inverse operations:
24 – 6 is 18, then halve which is 9.
Jack started with 9; check by substituting.

14 10 – third of a number = 6
This means that a third of the number is 4
The number must be 12.
Check by substitution:
10 – third of 12 is 6 ... correct.

15 48 ÷ __ + 5 = 13 means 48 ÷ ___ = 8
This means the missing number is 6,
as 48 ÷ 6 = 8.

16 93, 98, 103, 108, 113, ...
This means the missing number is 113.

17 The number of dots is 1, 5, 9, ...
The pattern is counting forward by 4
The next number is 13.

18 The pattern is 1, 5, 9, 13, 17, 21, ...
As 21 is the 6th term, there are 21 dots in Fig 6.

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1 C **2** B **3** D **4** 102 **5** A **6** D **7** D **8** C **9** B **10** A **11** C **12** B **13** B **14** A **15** B **16** D

EXPLANATIONS

1 Matches = 2 × triangles + 1
This means 2 × 6 + 1 = 13.
[We could have extended the pattern:
3, 5, 7, 9, 11, 13]

2 From question 1, Matches = 2 × triangles + 1
then 2 × 10 + 1 = 21
You need 21 matches to make 10 triangles.

3 Again, from question 1,
Matches = 2 × triangles + 1
then 2 × 50 + 1 = 101

ANSWERS
Week 3

4. $7 \times \Delta = 714$, means $\Delta = 714 \div 7$
$= 102$
The missing number is 102.

5. Work backwards from Suzie's answer:
32 subtract 8 is 24, then divide by 4 is 6
Suzie's number was 6.
[We can check this answer by substituting 6 back into the question]
We could also have tried each of the choices to find the number.

6. Pattern is counting forwards by 8
This means 25, 33, 41, 49, 57, 65, ...
The missing number is 65.

7. Using inverse operations, Amaya's number will be found by the rule: 'Add 5 to 17 and then divide by 2.'

8. Pattern is counting forwards by 6
This means 21, 27, 33, 39, 45, 51, 57
The number is 51.

9. Sean uses the rule 'multiply top number by 3 and add 2'. Check some of the numbers:
$6 \times 3 + 2 = 20$; $11 \times 3 + 2 = 35$, etc.

10. If $\triangle = 6$, then $6 + 6 + \bigcirc = 15$, or
$12 + \bigcirc = 15$ means $\bigcirc = 3$

11. Use inverse operations: 3 times 8 is 24, then divide by 6 is 4.
Gavin started with the number 4.

12. The sequence is the square numbers:
1, 4, 9, 16, 25, 36, 49
This means the missing numbers are 9 and 16.

13. As $45 = 2 \times \_\_ + 5$ means $40 = 2 \times \_\_$
The missing number is 20.

14. From 64, subtract 8 gives 56 and then divide by 8 which is 7.
Shari started with the number 7.

15. The pattern is 0.3, 0.8, 1.3, 1.8, ...

16. Using the rule :'bottom number = 30 minus 2 times top number' means
$X = 30 - 2 \times 5$
$= 30 - 10$
$= 20$ This means X = 20.

MEASUREMENT AND GEOMETRY (Test Your Skills)
2D and 3D shapes and position Page 76

1. The angle is obviously more than a right angle (90°) and less than a straight angle (180°). The angle is about 120°.

2. $x = 180 - 50$ (straight angle = 180°)
$= 130$

3. $x = 180 - (100 + 30)$ (angles in $\Delta = 180°$)
$= 180 - 130$
$= 50$

4. With angles of 100°, 30° and 50°, the triangle is scalene.

5. All angles equal and therefore equal to 60°.

6. A rhombus has all sides equal and the diagonals are of different lengths.

7. A quadrant is not shown on the diagram.

8. An isosceles triangle has 1 axis of symmetry.

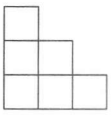
9. An equilateral triangle has an order of rotational symmetry of 3.

10. The sides have been doubled in size.
This means $x = 2.3 \times 2$
$= 4.6$

11. As $3 \times 15 = 45$, diameter is 45 cm.

12. With 5 faces, 9 edges and 6 vertices, the shape is a triangular prism.

13. The cross-section would be a rectangle.

14. 

15. The net consists of 2 triangles and 3 rectangles.

16. R is south-east of J.

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1 C **2** B **3** D **4** D **5** D **6** B **7** C **8** A **9** B **10** B
11 A **12** B **13** A **14** A **15** C **16** D

NUMBER AND ALGEBRA
Patterns and algebra

1 65, 78, 91, ____
What is the missing value?
A 13 B 103 C 104 D 106

2 $\frac{2}{5}$, $\frac{4}{5}$, ____, $1\frac{3}{5}$, 2
What is the missing value?
A $\frac{1}{5}$ B $\frac{2}{5}$ C 1 D $1\frac{1}{5}$

3 0.2, 0.9, 1.6, ____ , ____ , 3.7
What number goes here?
A 0.7 B 2.3 C 3 D 3.2

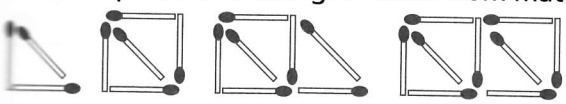
4 16, 12, 8, 4, ...
What is the sixth number in the pattern?
A –12 B –8 C –4 D 0

5

Top	3	5	9
Bottom	X	17	25

The rule used to complete the table is 'bottom number is double the top number and then add 7'. What is the value of X?
A 11 B 12 C 13 D 14

Here is a pattern of triangles made from matches:



6 How many matches are needed to make 5 triangles?
A 10 B 11 C 12 D 15

7 The diagrams are used to complete a table.

Triangles	1	2	3	4	5	6	7	8	9
Matches	3	5	7	9				X	

What is the value of X?
A 13 B 16 C 17 D 24

8

Top	12	17	22	X	32
Bottom	48	43	38	Y	28

Which is correct?
A X = 27 and Y = 43 B X = 23 and Y = 35
C X = 27 and Y = 35 D X = 27 and Y = 33

The pattern of dots continues.

9 How many dots would be in the 5th figure?
A 11 B 12 C 13 D 15

10 A table summarises the figures and dots.

Figure	1	2	3	4	5	6
Dots	1	3	6	10		X

What is the value of X?
A 15 B 16 C 18 D 21

11 295 – ☐ = 86
What is the missing number?
A 209 B 211 C 219 D 229

12 4 × ☐ – 2 = 18
What is the missing number?
A 5 B 4 C 2 D 10

13 Jack is thinking of a number. He doubles it and adds six. His answer is twenty-four. What was Jack's original number?
A 6 B 9 C 12 D 16

14 A third of what number subtracted from ten is the same as six?
A 6 B 9 C 12 D 15

15 48 ÷ ☐ + 5 = 13
What is the missing number?
A 6 B 8 C 18 D 30

16 Lee started at 93 and counted forward by 5. His second number was 98. What is his fifth number?
A 5 B 103 C 108 D 113

Here is a pattern of dots:

17 How many dots will be in Fig 4?
A 13 B 14 C 15 D 16

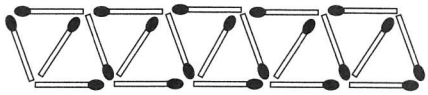
18 Which figure will consist of 21 dots?
A Fig 5 B Fig 6 C Fig 7 D Fig 8

Explanations on page 212

Answers: 1 C 2 D 3 B 4 C 5 C 6 B 7 C 8 D 9 D 10 D 11 A 12 A 13 B 14 C 15 A 16 D 17 A 18 B

NUMBER AND ALGEBRA
Patterns and algebra

Arjun is making this pattern of triangles using matches:



He summarised the information in the table:

Triangle	1	2	3	4	5	6
Matches	3	5	7			X

1 What number replaces the X? *Hint 1*
A 9 B 12 C 13 D 18

2 How many matches are needed to make 10 triangles?
A 20 B 21 C 26 D 30

3 If Arjun continues the pattern, how many matches would he need for 50 triangles?
A 51 B 81 C 86 D 101

4 What is the missing number in this number sentence? $7 \times \Delta = 714$ *Hint 2*
Write your answer in the box: ☐

5 Suzie was thinking of a number. She said that if you multiplied the number by 4 and then added 8, the answer would be 32. What was Suzie's number? *Hint 3*
A 6 B 8 C 12 D 28

6 Kali wrote this number pattern.
25, 33, 41, 49, ...
If the rule remained the same, what was Kali's sixth number in the pattern?
A 8 B 48 C 55 D 65

7 Amaya thought of a number. When she doubled her number and subtracted 5, her answer was 17. Which of the following statements shows the correct way to work out Amaya's number?
A Divide 17 by 2 and add 5
B Multiply 17 by 2 and add 5
C Add 5 to 17 and then multiply by 2
D Add 5 to 17 and then divide by 2

8 This pattern shows some missing numbers.
21, 27, 33, 39, ___, ___, 57
↑
What number goes here?
A 6 B 49 C 51 D 55

9 Sean used the same rule on each number in the top row to get the number below it in the bottom row. What rule did Sean use?

Top	6	11	16	21
Bottom	20	35	50	65

A add 14 to the top number
B multiply top number by 3 and add 2
C multiply top number by 4 and minus 4
D multiply top number by 5 and minus 10

10 Look at this number sentence:
$\triangle + \triangle + \bigcirc = 15$
If $\triangle = 6$, what is the value of the $\bigcirc$?
A 3 B 4 C 5 D 6

11 Gavin starts with a number. He multiplies it by 6 and then divides by 8. His answer is 3. What number did Gavin start with?
A 2 B 3 C 4 D 6

12 1, 4, ___, ___, 25, 36, 49
What are the missing numbers?
A 5 and 10 B 9 and 16
C 10 and 15 D 7 and 10

13 If $45 = 2 \times$ ☐ $+ 5$, what is the missing value? *Hint 4*
A 15 B 20 C 25 D 30

14 Shari thought of a number. She multiplied it by 8 and then added 8. Her answer was 64. What number did Shari start with?
A 7 B 8 C 9 D 16

15 Start with 0.3 and count forward by 0.5. Which pattern shows this rule?
A 0.3, 0.5, 0.7, 0.9, ... B 0.3, 0.8, 1.3, 1.8, ...
C 0.3, 0.8, 1.2, 1.6, ... D 0.3, 0.8, 1.2, 1.7, ...

16 Teresa used the rule 'Bottom number = 30 minus 2 times top number' to complete the table. What is the value of X in the table?

Top	3	5	11	13
Bottom	24	X	Y	4

A 7 B 8 C 10 D 20

Hint 1: Use the first 3 numbers to develop a rule and then apply it.
Hint 2: The inverse operation of multiplication is division.
Hint 3: Sometimes it is easier to substitute the choices into the question to find the correct one.
Hint 4: The inverse operation of addition is subtraction.

☞ Answers and explanations on pages 212-213