Activity

The matrices below are used in the questions below:

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \quad B=\begin{bmatrix}-1 & 2 & 0 & 4\end{bmatrix} \quad C=\begin{bmatrix}0\end{bmatrix} \quad D=\begin{bmatrix}1\\ 6\end{bmatrix} \quad E=\begin{bmatrix}3 & -1\\ 0 & -2\\ 1 & 5\end{bmatrix}$$

$$F=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \quad G=\begin{bmatrix}1 & 0\\ 0 & 8\end{bmatrix} \quad H=\begin{bmatrix}3 & 0 & 1\\ -1 & -2 & 5\end{bmatrix} \quad J=\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$$

1. State the size of each matrix.

2. Which of the matrices are row matrices?

3. Which of the matrices are column matrices?

4. Which of the matrices are zero matrices?

5. Which of the matrices are square matrices?

6. Which of the matrices is an identity matrix?

7. If $\begin{bmatrix}a & b+1 & 9\\ 3c & 0 & d-2\end{bmatrix}=\begin{bmatrix}1 & 3 & 9\\ 12 & 0 & 0\end{bmatrix}$ determine the values of *a*, *b*, *c* and *d*.

8. Determine A^T, E^T and J^T

9. Compute the following using the matrices above;
   (i) 3E (ii) 2A + G (iii) G – 2J

10. Perform the following multiplications if it is possible.
    (i) GH (ii) GD (iii) AJ (iv) DG

11. The solution to the system of linear equations below is $x=6$ and $y=4$.

$$x+y=10$$
$$x-y=2$$

Obtain the matrices *A*, *X* and *B*. Verify that $AX=B$

12. You are told that the inverse of $\begin{bmatrix}8 & 5\\ 3 & 2\end{bmatrix}$ is $\begin{bmatrix}2 & -5\\ -3 & 8\end{bmatrix}$. Confirm this is correct or otherwise by performing a multiplication.

13. Using the first method, find the inverse of each of the matrices below, if possible;

(i) $\begin{bmatrix} -1 & 5 \\ 1 & -3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -4 \\ 0 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 12 & -6 \\ -4 & 2 \end{bmatrix}$

14. Using the Gauss Jordan Method, find the inverse of each of the matrices below, if possible;

(i) $\begin{bmatrix} -2 & 5 \\ 2 & -4 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & -5 \\ 4 & 10 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 5 \\ -1 & 2 \end{bmatrix}$

15. Solve the system of linear equations below by using Matrices.

$$x - 3y = -11$$
$$4x + 3y = 9$$

16. Solve the system of linear equations below by using Matrices.

$$2x - 5y = 1$$
$$8x - 20y = 4$$

17. Using the matrices given at the beginning, calculate the matrix A^2 and J^2

18. In Algebra, the difference of squares identity is $(a-b)(a+b) = a^2 - b^2$.

Using the Matrices $P = \begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix}$ *and* $Q = \begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}$ determine if the same relationship may exist for matrices, ie: $(P-Q)(P+Q) = P^2 - Q^2$

Answers to Activity questions

The matrices below are used in the questions below:

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\quad B=\begin{bmatrix}-1 & 2 & 0 & 4\end{bmatrix}\quad C=\begin{bmatrix}0\end{bmatrix}\quad D=\begin{bmatrix}1\\ 6\end{bmatrix}\quad E=\begin{bmatrix}3 & -1\\ 0 & -2\\ 1 & 5\end{bmatrix}$$

$$F=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\quad G=\begin{bmatrix}1 & 0\\ 0 & 8\end{bmatrix}\quad H=\begin{bmatrix}3 & 0 & 1\\ -1 & -2 & 5\end{bmatrix}\quad J=\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$$

1. State the size of each matrix.

$$A{:}2\times 2\ \ B{:}\ 1\times 4\ \ C{:}1\times 1\ \ D{:}2\times 1\ \ E{:}3\times 1\ \ F{:}3\times 3\ \ G{:}2\times 2\ \ H{:}2\times 3\ \ J{:}2\times 2$$

2. Which of the matrices are row matrices? B, C

3. Which of the matrices are column matrices? C, D

4. Which of the matrices are zero matrices? C

5. Which of the matrices are square matrices? A, C, F, G, J

6. Which of the matrices is an identity matrix? F

7. If $\begin{bmatrix}a & b+1 & 9\\ 3c & 0 & d-2\end{bmatrix}=\begin{bmatrix}1 & 3 & 9\\ 12 & 0 & 0\end{bmatrix}$ determine the values of *a*, *b*, *c* and *d*.

$$a=1,\quad b+1=3\rightarrow b=2,\quad 3c=12\rightarrow c=4,\quad d-2=0\rightarrow d=2$$

8. Determine A^T, E^T *and* J^T

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\rightarrow A^T=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$$

$$E=\begin{bmatrix}3 & -1\\ 0 & -2\\ 1 & 5\end{bmatrix}\rightarrow E^T=\begin{bmatrix}3 & 0 & 1\\ -1 & -2 & 5\end{bmatrix}$$

$$J=\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}\rightarrow J^T=\begin{bmatrix}4 & -3\\ -2 & 1\end{bmatrix}$$

9. Compute the following using the matrices above;

(i) 3E $$3E = 3\begin{bmatrix} 3 & -1 \\ 0 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ 0 & -6 \\ 3 & 15 \end{bmatrix}$$

(ii) 2A + G $$2A + G = 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 6 & 16 \end{bmatrix}$$

(iii) G – 2J

$$G - 2J = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix} - 2\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -6 & 2 \end{bmatrix} = \begin{bmatrix} -7 & 4 \\ 6 & 6 \end{bmatrix}$$

10. Perform the following multiplications if it is possible.

(i) GH

$$GH = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 \\ -1 & -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times 3)+(0\times -1) & (1\times 0)+(0\times -2) & (1\times 1)+(0\times 5) \\ (0\times 3)+(8\times -1) & (0\times 0)+(8\times -2) & (0\times 1)+(8\times 5) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 1 \\ -8 & -16 & 40 \end{bmatrix}$$

(ii) GD

$$GD = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix} = \begin{bmatrix} (1\times 1)+(0\times 6) \\ (0\times 1)+(8\times 6) \end{bmatrix} = \begin{bmatrix} 1 \\ 48 \end{bmatrix}$$

(iii) AJ $$AJ = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} (1\times 4)+(2\times -3) & (1\times -2)+(2\times 1) \\ (3\times 4)+(4\times -3) & (3\times -2)+(4\times 1) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

(iv) DG this multiplication cannot be performed.

11. The solution to the system of linear equations below is *x=6* and *y=4*.

$$x + y = 10$$

$$x - y = 2$$

Obtain the matrices *A*, *X* and *B*. Verify that *AX*=*B*

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 6 \\ 4 \end{bmatrix} \quad B = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$$

$$AX = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 6+4 \\ 6-4 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix} = B$$

12. You are told that the inverse of $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ is $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$. Confirm this is correct or otherwise by performing a multiplication.

$$\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 16-15 & -40+40 \\ 6-6 & -15+16 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

The matrices are inverses.

13. Using the first method, find the inverse of each of the matrices below, if possible;

(i) $\begin{bmatrix} -1 & 5 \\ 1 & -3 \end{bmatrix}$ $\qquad A^{-1} = \dfrac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \dfrac{1}{3-5}\begin{bmatrix} -3 & -5 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{5}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & -4 \\ 0 & 1 \end{bmatrix}$ $\qquad A^{-1} = \dfrac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \dfrac{1}{2}\begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 2 \\ 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 12 & -6 \\ -4 & 2 \end{bmatrix}$ $\qquad det(A) = (12 \times 2) - (-6 \times -4) = 24 - 24 = 0$

It is not possible to find the inverse.

14. Using the Gauss Jordan Method, find the inverse of each of the matrices below, if possible;

(i) $\begin{bmatrix} -2 & 5 \\ 2 & -4 \end{bmatrix}$

The first step is to obtain a '1' for the element $a_{1,1}$. This can be performed by dividing the first row by -2.	$\left[\begin{array}{cc\|cc} -2 & 5 & 1 & 0 \\ 2 & -4 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc\|cc} 1 & \frac{-5}{2} & \frac{-1}{2} & 0 \\ 2 & -4 & 0 & 1 \end{array}\right]$
The next step is to get a '0' for the element $a_{2,1}$. Multiply Row 1 by -2 and add to Row 2.	$\left[\begin{array}{cc\|cc} 1 & \frac{-5}{2} & \frac{-1}{2} & 0 \\ 2 & -4 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc\|cc} 1 & \frac{-5}{2} & \frac{-1}{2} & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$
The next step is to get a '1' in cell $a_{2,2}$ and then use this to get a '0' for element $a_{1,2}$. As there is a '1' already, nothing is required	$\left[\begin{array}{cc\|cc} 1 & \frac{-5}{2} & \frac{-1}{2} & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$
Multiply Row 2 by $^5/_2$ and add to Row 1	$\left[\begin{array}{cc\|cc} 1 & \frac{-5}{2} & \frac{-1}{2} & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc\|cc} 1 & 0 & 2 & \frac{5}{2} \\ 0 & 1 & 1 & 1 \end{array}\right]$
Now this is in the form of $\left[I \mid A^{-1} \right]$, so	$A^{-1} = \begin{bmatrix} 2 & \frac{5}{2} \\ 1 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} -1 & -5 \\ 4 & 10 \end{bmatrix}$

The first step is to obtain a '1' for the element $a_{1,1}$. This can be performed by multiplying the first row by -1.	$\left[\begin{array}{cc\|cc} -1 & -5 & 1 & 0 \\ 4 & 10 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc\|cc} 1 & 5 & -1 & 0 \\ 4 & 10 & 0 & 1 \end{array}\right]$
The next step is to get a '0' for the element $a_{2,1}$. Multiply Row 1 by -4 and add to Row 2.	$\left[\begin{array}{cc\|cc} 1 & 5 & -1 & 0 \\ 4 & 10 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc\|cc} 1 & 5 & -1 & 0 \\ 0 & -10 & 4 & 1 \end{array}\right]$
The next step is to get a '1' in cell $a_{2,2}$ and then use this to get a '0' for element $a_{1,2}$. Row 2 is divided by -10	$\left[\begin{array}{cc\|cc} 1 & 5 & -1 & 0 \\ 0 & -10 & 4 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc\|cc} 1 & 5 & -1 & 0 \\ 0 & 1 & \frac{-2}{5} & \frac{-1}{10} \end{array}\right]$

Multiply Row 2 by -5 and add to Row 1	$\left[\begin{array}{cc\|cc} 1 & 5 & -1 & 0 \\ 0 & 1 & \frac{-2}{5} & \frac{-1}{10} \end{array}\right] \to \left[\begin{array}{cc\|cc} 1 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & \frac{-2}{5} & \frac{-1}{10} \end{array}\right]$
Now this is in the form of $\left[I \mid A^{-1} \right]$, so	$A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{-2}{5} & \frac{-1}{10} \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 5 \\ -1 & 2 \end{bmatrix}$

The first step is to obtain a '1' for the element $a_{1,1}$. This can be performed by dividing the first row by 2.	$\left[\begin{array}{cc\|cc} 2 & 5 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc\|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ -1 & 2 & 0 & 1 \end{array}\right]$
The next step is to get a '0' for the element $a_{2,1}$. Multiply Row 1 by 1 and add to Row 2.	$\left[\begin{array}{cc\|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ -1 & 2 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc\|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ 0 & \frac{9}{2} & \frac{1}{2} & 1 \end{array}\right]$
The next step is to get a '1' in cell $a_{2,2}$ and then use this to get a '0' for element $a_{1,2}$. Row 2 is multiplied by $^2/_9$	$\left[\begin{array}{cc\|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ 0 & \frac{9}{2} & \frac{1}{2} & 1 \end{array}\right] \to \left[\begin{array}{cc\|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{9} & \frac{2}{9} \end{array}\right]$
Multiply Row 2 by $-^5/_2$ and add to Row 1	$\left[\begin{array}{cc\|cc} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{9} & \frac{2}{9} \end{array}\right] \to \left[\begin{array}{cc\|cc} 1 & \frac{5}{2} & \frac{2}{9} & \frac{-5}{9} \\ 0 & 1 & \frac{1}{9} & \frac{2}{9} \end{array}\right]$
Now this is in the form of $\left[I \mid A^{-1} \right]$, so	$A^{-1} = \begin{bmatrix} \frac{2}{9} & \frac{-5}{9} \\ \frac{1}{9} & \frac{2}{9} \end{bmatrix}$

15. Solve the system of linear equations below by using Matrices.

$$x - 3y = -11$$

$$4x + 3y = 9$$

$$A = \begin{bmatrix} 1 & -3 \\ 4 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} -11 \\ 9 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} -11 \\ 9 \end{bmatrix}$$
$$= \frac{1}{15}\begin{bmatrix} 3 & 3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} -11 \\ 9 \end{bmatrix}$$
$$= \frac{1}{15}\begin{bmatrix} -33+27 \\ 44+9 \end{bmatrix}$$
$$= \frac{1}{15}\begin{bmatrix} -6 \\ 53 \end{bmatrix}$$
$$= \begin{bmatrix} -0.4 \\ 3.5\dot{3} \end{bmatrix}$$

16. Solve the system of linear equations below by using Matrices.

$$2x - 5y = 1$$
$$8x - 20y = 4$$
$$A = \begin{bmatrix} 2 & -5 \\ 8 & -20 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$det(A) = 0$ no inverse can be found. You will notice that one equation is a constant multiple of the other. Graphically they result in the same line. Any solution from the line will solve both equations, so there are an infinite number of solutions.

17. Using the matrices given at the beginning, calculate the matrix A^2 and J^2

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$
$$J^2 = J \times J = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 22 & -10 \\ -15 & 7 \end{bmatrix}$$

18. In Algebra, the difference of squares identity is $(a-b)(a+b) = a^2 - b^2$.

Using the Matrices $P = \begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix}$ $and\ Q = \begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}$ determine if the same relationship may exist for matrices, ie: $(P-Q)(P+Q) = P^2 - Q^2$

$$P - Q = \begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -2 & -2 \end{bmatrix}$$
$$P + Q = \begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 2 & 0 \end{bmatrix}$$
$$(P-Q)(P+Q) = \begin{bmatrix} 5 & 3 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} -1 & 5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 25 \\ -2 & -10 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 1 \end{bmatrix}$$

$$Q^2 = \begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 11 & -2 \\ -4 & 3 \end{bmatrix}$$

$$P^2 - Q^2 = \begin{bmatrix} 4 & 4 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 11 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -7 & 6 \\ 4 & -2 \end{bmatrix}$$

This shows that

$$(P-Q)(P+Q) \neq P^2 - Q^2$$