Activity

1. Solve the following Logarithmic Equations.

(a)	$\log_{10} x = 2$	(b)	$\log_5 x = -2$
(c)	$\log_4 x = 0.5$	(d)	$\log_7 2x = 2$
(e)	$\log_2 (x+4) = 3$	(f)	$\log_9 \left(\frac{x}{4}\right) = -0.5$
(g)	$\log_3 x^2 = 2$	(h)	$\log_3 x = 2$
(i)	$\log_2 (x^2 + 2x) = 3$	(j)	$\log_x 4 = 4$
(k)	$\log_x 9 = 0.5$	(l)	$\log_x 64 = 2$
(m)	$\log_x (2x-1) = 2$	(n)	$\log_x 3 = 2$
(o)	$\log_x 10 = 3$	(p)	$\log_4 64 = x$
(q)	$10^{\log x} = 4$	(r)	$e^{\ln(x+4)} = 7$

2. Solve for x in the Logarithmic Equations.

(a)	$2\log x = \log 4$	(b)	$2\ln 2x = \ln 36$
(c)	$\log(x+5) = \log x + \log 6$	(d)	$3\log x = \log 4$
(e)	$2\log x = \log 2x + \log 3$	(f)	$2\log_3 2 - \log_3(x+1) = \log_3 5$
(g)	$\log_4 x + \log_4 6 = \log_4 12$	(h)	$3\log_3 2 + \log_3(x-2) = \log_3 5$
(i)	$\log_8(x+2) = 2 - \log_8 2$	(j)	$\log_2 x = 5 - \log_2(x+4)$
(k)	$\log x + \log 3 = \log 5$	(l)	$\log x - \log(x-1) = \log 4$
(m)	$\log(x-3) = 3$	(n)	$\ln(4-x) + \ln 2 = 2\ln x$
(o)	$\log_4(2x+4) - 3 = \log_4 3$	(p)	$\log_2 x + 3\log_2 2 = \log_2 \frac{2}{x}$

3. Solve for y in terms of the other variables present.

(a)	$\log y = \log x + \log 4$	(b)	$\log y = \frac{1}{2}\log 5 + \frac{1}{2}\log x$
(c)	$\log y = \log 4 - \log x$	(d)	$\log y + \log x = \log 4 + 2$
(e)	$\log_5 x + \log_5 y = \log_5 3 + 1$	(f)	$2\ln y = \ln e^2 - 3\ln x$
(g)	$\log_2 y + 2\log_2 x = 5 - \log_2 4$	(h)	$\log_7 y = 2\log_7 5 + \log_7 x + 2$
(i)	$3\ln y = 2 + 3\ln x$	(j)	$2(\log_4 y - 3\log_4 x) = 3$
(k)	$2^y = e^x$	(l)	$10^y = 3^{x+1}$

Exponential Equations

An equation where the exponent (index) is a variable or contains a variable is called an exponential equation. An example is $3^x = 20$.

Let's consider the example below:

If an investor deposits $20 000 in an account that compounds at 5% p.a., how long is it before this amount has increased to $30 000? (Compounded annually)

Using the compound interest formula gives:

$$A = P(1+i)^n$$

$$30000 = 20000(1.05)^n$$

The first step is to rearrange the equation to get the form $a = b^x$ or $b^x = a$

$$\frac{30000}{20000} = 1.05^n$$

$$1.5 = 1.05^n$$

Then take the log of both sides. Because calculations will need to be performed, ordinary logarithms or natural logarithms should be used.

$1.5 = 1.05^n$	$1.5 = 1.05^n$
$\log 1.5 = \log 1.05^n$ taking the log of both sides	$\ln 1.5 = \ln 1.05^n$ taking the log of both sides
$\log 1.5 = n \log 1.05$ using the third log law	$\ln 1.5 = n \ln 1.05$ using the third log law
$\frac{\log 1.5}{\log 1.05} = n$ rearranging	$\frac{\ln 1.5}{\ln 1.05} = n$ rearranging
$n = 8.31$	$n = 8.31$

This means that it will take 8.31years (9 years in practice) for the growth in value to occur.

This can be checked by calculating:

$$1.05^{8.31}$$

$$= 1.5$$

This example contains a simple power.

$$3^x = 10$$

$$\log 3^x = \log 10$$

$$x \log 3 = \log 10$$

$$x = \frac{\log 10}{\log 3}$$

$$x = \frac{1}{\log 3}$$

$$x = 2.096$$

In this example, the 4 must be rearranged to obtain the power as the subject before taking the logarithm of both sides.

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$$4 \times 7^x = 20$$
$$7^x = \frac{20}{4}$$
$$7^x = 5$$
$$\log 7^x = \log 5$$
$$x \log 7 = \log 5$$
$$x = \frac{\log 5}{\log 7}$$
$$x = 0.8271$$

In this example, there is a power as the subject; however, the exponent is more complex than previous examples.

$$12.5^{2n-1} = 523.95$$
$$\log 12.5^{2n-1} = \log 523.95$$
$$(2n-1)\log 12.5 = \log 523.95$$
$$2n-1 = \frac{\log 523.95}{\log 12.5}$$
$$2n-1 = 2.479$$
$$2n = 3.4790$$
$$n = 1.7395$$

Example: Joan wants to retire when her superannuation fund reaches $500 000. She invests $1500 a month (after tax) into her superannuation fund. She assumes that the superannuation fund will return 6%pa or 0.5%pm. How long before her goal is reached?

This is a Future Value of an Annuity, with r = 0.005, R = 1500 and S = 500000.

$$S = R\frac{\left[(1+r)^n - 1\right]}{r}$$
$$500000 = 1500\frac{\left[(1+0.005)^n - 1\right]}{0.005}$$
$$2500 = 1500\left[(1+0.005)^n - 1\right] \quad \text{•mult both sides by 0.005}$$
$$1.666\overline{6} = 1.005^n - 1 \quad \text{•divide both sides by 1500}$$
$$2.666\overline{6} = 1.005^n \quad \text{•add 1 to both sides}$$

To solve this exponential equation, logs are required.

$$2.666\overline{6} = 1.005^n$$

$$\log 2.666\overline{6} = \log 1.005^n \quad \text{•take Log of both sides}$$

$$\log 2.666\overline{6} = n\log 1.005 \quad \text{•use Log Law 3}$$

$$\frac{\log 2.666\overline{6}}{\log 1.005} = n$$

$$n = 197 \text{ (rounded up)}$$

It will be 197 months or 16 years 5 months before Joan has enough money.

Video 'Solving Exponential Equations'

Activity

1. Solve the following exponential equations.

(a) $4^x = 20$

(b) $1.6^x = 3.2$

(c) $3^{2x} = 0.125$

(d) $4.6^{0.5x} = 100$

(e) $6^{x+1} = 40$

(f) $11.2^{2-x} = 0.6$

(g) $2^x + 12 = 40$

(h) $3 \times 6^x - 10 = 12.5$

(i) $6^{x+4} = 6^{11}$

(j) $5^x = 2^{2x+1}$

2. If an investor deposits \$250 000 in an account that compounds at 7% p.a., how long is it before this amount has increased to \$400 000? (Compounded semi-annually). Use the compound interest formula: $A = P(1+i)^n$