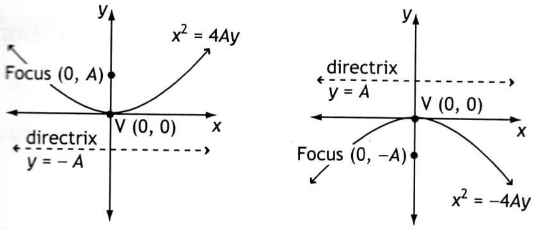
Equations reducible to quadratics

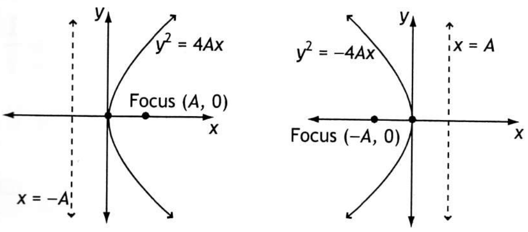
Suitable substitution reduces equations to quadratics.

The parabola $x^2 = \pm 4Ay$

- Vertex at origin, focal length A

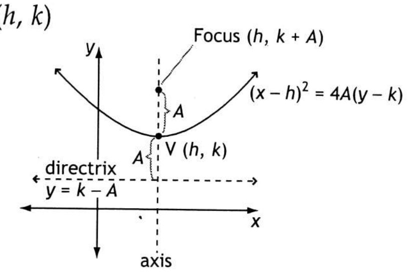


- Of lesser importance is the parabola $y^2 = \pm 4Ax$



The parabola $(x - h)^2 = \pm 4A(y - k)$

Vertex at (h, k)



Finding focal length of $y = ax^2 + bx + c$

Complete the square and put into the form

$$(x - h)^2 = 4A(y - k)$$

1.6 Plane geometry

See Test Yourself questions 67–70 at the end of this chapter.

Properties of quadrilaterals

- **Parallelogram**
  - Opposite sides and angles are equal.
  - Diagonals bisect each other.
  - Both pairs of opposite sides parallel.
- **Rhombus**
  - Has all the properties of a parallelogram.
  - Diagonals bisect each other at right angles.
  - Diagonals bisect angles through which they pass.
- **Rectangle**
  - Has all the properties of a parallelogram.
  - One angle is a right angle.
- **Square**
  - Has all the properties of a rectangle.
  - A pair of adjacent sides are equal.

Tests for special quadrilaterals

- **Parallelogram**

  A quadrilateral is a parallelogram if:
  - two opposite sides are equal and parallel, or
  - both pairs of opposite sides are equal, or
  - both pairs of opposite sides are parallel, or
  - opposite angles are equal, or
  - diagonals bisect each other.
- **Rhombus**

  A quadrilateral is a rhombus if:
  - all sides are equal, or
  - diagonals bisect each other at right angles.
- **Rectangle**

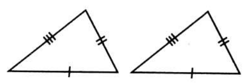
  A quadrilateral is a rectangle if the diagonals are equal.
- **Square**

  To prove that a quadrilateral is a square, show that it is a rectangle with a pair of adjacent sides equal.

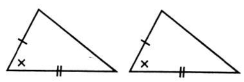
Congruent triangles

Two triangles are congruent if they are equal in all respects, i.e. corresponding sides and angles of two congruent triangles are equal. To prove that a pair of triangles are congruent you use one of the following four tests.

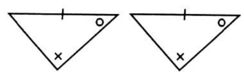
1 **SSS test:** three sides of one triangle respectively equal to three sides of the other.



2 **SAS test:** two sides and an **included angle** of one triangle respectively equal to two sides and the included angle of the other.

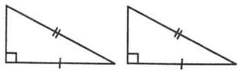


3 **AAS test:** two angles of one triangle are equal to two angles of the other and the **corresponding sides** are equal.



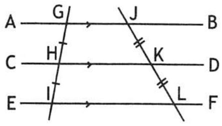
To Test Yourself go to page 12

4 **RHS test:** in the case of a right-angled triangle, if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other, the triangles are congruent.



Intercept properties of transversals to parallel lines

- If three or more parallel lines are cut by two transversals, the intercepts formed on each transversal are in the same ratio.



$$\frac{GH}{HI} = \frac{JK}{KL}$$

- It follows that if a family of parallel lines cut equal intercepts on one transversal, it does so on all transversals.

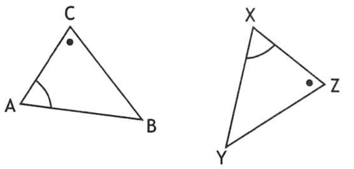
Similar triangles

To prove that a pair of triangles are similar, you must use one of the following tests.

1 If the **angles** of one triangle are respectively equal to the corresponding angles of the other triangle, then the triangles are similar.

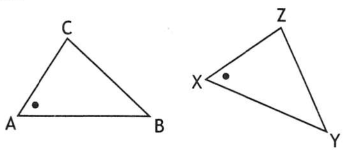
We need only to show that two angles of one triangle are respectively equal to two angles of the other. As the angle sum of any triangle is 180°, the third angle in each triangle will thus be equal.

2 If the **corresponding sides** of two triangles are in proportion (same ratio) then the triangles are similar.



i.e. $\Delta ABC \,|||\, \Delta XYZ$ if $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

3 If **two sides** of each triangle are in proportion and the **included angles** are equal, then the triangles are similar.



i.e. $\frac{AC}{XZ} = \frac{AB}{XY}$ and $\angle A = \angle X$, then $\Delta ABC \,|||\, \Delta XYZ$

To Test Yourself go to pages 12–13

1.7 The tangent to a curve and the derivative of a function

See Test Yourself questions 71–80 at the end of this chapter.

Limits

- $\lim_{x \to a} x^2 - 4x + 1$ substitute $x = a$ in expression
- $\lim_{x \to a} \frac{x^2 + \ldots}{x - a}$ factorise numerator so that factor $(x - a)$ cancels out
- $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ divide numerator and denominator by highest power of x and apply rule $\lim_{x \to \infty} \frac{1}{x} = 0$; similarly $\frac{1}{x^2}$, etc.]

Secant

- The secant lies between (x_1, y_1) and (x_2, y_2).
- Gradient $= m = \frac{y_2 - y_1}{x_2 - x_1}$
- Equation: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

Derivative

- Derivative is a measure of the **gradient of a curve** at any point, and hence the gradient of the tangent at any point.
- Notation: $f'(x)$, y', $\frac{dy}{dx}$, $\frac{d}{dx}(f(x))$
- If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

Equation of tangent

Use $y - y_1 = m(x - x_1)$, where m = gradient found by differentiating.

Gradient of normal

A normal is a line perpendicular to a tangent at a point of contact.

∴ gradient of normal
= negative reciprocal of gradient of tangent