KEYWORDS

Consecutive	**Rationals**
Equation	**Solution**
Integers	**Substitution**

An equation is an algebraic expression (sentence) with an equals sign. That is, a sentence like $x + 3 = 7$.

Solving an equation means finding the value of the pronumeral that makes the sentence true.

That is, if $x + 3 = 7$

Then $x = 4$

This is the solution of the equation.

Solution of Equations

One-Step Equations

When solving equations we use the following rules.

Rule 1

$x + a = b$ ('+' = '−')

$\therefore x = b - a$

The $+ a$ on the left-hand side of the equals sign becomes $- a$ when it goes over to the right-hand side of the equal sign.

For Example

1 Solve $x + 4 = 12$.

1 $x + 4 = 12$

$\therefore x = 12 - 4$

$\therefore x = 8$

The + 4 goes over the equals sign and becomes − 4.

Rule 2

$x - a = b$ ('−' = '+')

$\therefore x = b + a$

The $-a$ on the left-hand side of the equals sign becomes $+a$ when it goes over to the right-hand side of the equals sign.

For Example

1 Solve $a - 3 = 5$.

1 $a - 3 = 5$

$\therefore a = 5 + 3$

$\therefore a = 8$

The − 3 goes over the equals sign and becomes + 3.

Rule 3

$ax = b$ ('×' = '÷')

$\therefore x = \frac{b}{a}$

This means '$a \times x$'.

The a on the left-hand side goes over the equals sign and divides the right-hand side. It does not change sign.

For Example

1 Solve $3x = 12$.

1 $3x = 12$

$\therefore x = \frac{12}{3}$

$\therefore x = 4$

The 3 goes over the equal sign and divides the 12.

Rule 4

$\frac{x}{a} = b$ ('$\div = \times$')

$x = ba$

This means '$x \div a$'.

The a on the left-hand side goes over the equal sign and multiplies the right-hand side.

For Example

1 Solve $\frac{x}{4} = 12$.

1 $\frac{x}{4} = 12$ ('$\div 4$' goes over the equal sign and multiplies the 12.)

$\therefore x = 12 \times 4$

$\therefore x = 48$

Examples Involving Rationals and Integers

The same rules apply.

For Example

Solve the following equations:

1 $x + 7 = 3$

2 $x - 3 = -8$

3 $4x = 12$

4 $x - \frac{1}{2} = \frac{3}{4}$

5 $x + 2\frac{1}{2} = 4\frac{3}{4}$

1 $x + 7 = 3$ (− over the equal sign)

$\therefore x = 3 - 7$

$\therefore x = -4$

[+ 7 goes over the equal sign and becomes − 7.]

2 $x - 3 = -8$ (+ over the equal sign)

$\therefore x = -8 + 3$

$\therefore x = -5$

[− 3 goes over the equal sign and becomes + 3.]

3 $4x = 12$

$\therefore x = \frac{12}{4}$

$\therefore x = 3$

[4 goes over the equal sign and divides 12. It does not become − 4.]

4 $x - \frac{1}{2} = \frac{3}{4}$ (+ over the equal sign)

$x = \frac{3}{4} + \frac{1}{2}$

$x = 1\frac{1}{4}$

[$-\frac{1}{2}$ goes over the equal sign and becomes $+\frac{1}{2}$.]

5 $x + 2\frac{1}{2} = 4\frac{3}{4}$ (− over the equal sign)

$x = 4\frac{3}{4} - 2\frac{1}{2}$

$x = 2\frac{1}{4}$

[$+2\frac{1}{2}$ goes over the equal sign and becomes $-2\frac{1}{2}$.]

Summary

- When a term goes over the equal sign of an equation, its sign is changed to the opposite sign.
- Each side of an equation may be multiplied or divided by the same number.

Two-Step Equations

This type of equation involves the steps of multiplying or dividing and adding or subtracting.

For Example

Solve the following equations:

1 $4x - 2 = 18$

2 $3x + 2 = 23$

3 $\frac{x - 2}{3} = 12$

1 $4x - 2 = 18$ [Taking -2 over to the other side of the equal sign gives $+2$.]

$\therefore 4x = 18 + 2$

$\therefore 4x = 20$

$\therefore x = \frac{20}{4}$ [Dividing by 4.]

$\therefore x = 5$

2 $3x + 2 = 23$ [Taking $+2$ over the equal sign gives -2.]

$\therefore 3x = 23 - 2$

$\therefore 3x = 21$

$\therefore x = \frac{21}{3}$ [Dividing by 3.]

$\therefore x = 7$

3 $\frac{x - 2}{3} = 12$

$\therefore x - 2 = 12 \times 3$ [Multiplying by 3.]

$\therefore x - 2 = 36$ [Taking -2 over the equal sign gives $+2$.]

$\therefore x = 36 + 2$

$\therefore x = 38$

Equations with Pronumerals on Both Sides of the Equals Sign

When solving an equation with pronumerals on both sides the pronumerals are moved to the left-hand side of the equal sign, and the numbers to the right-hand side.

For Example

Solve the following equations:

1 $7x - 3 = 5x + 11$

2 $4b - 3 = 3b + 6$

3 $5x + 4 = x + 34$

4 $4y = 21 - 3y$

1 $7x - 3 = 5x + 11$ [Taking -3 to the RHS gives $+3$ and $5x$ ($= +5x$) to the LHS gives $-5x$.]

$\therefore 7x - 5x = 11 + 3$ [Collecting like terms.]

$\therefore 2x = 14$

$\therefore x = \frac{14}{2}$ [Dividing by 2.]

$\therefore x = 7$

2 $4b - 3 = 3b + 6$

$\therefore 4b - 3b = 6 + 3$ [Collecting like terms.]

$\therefore b = 9$

3 $5x + 4 = x + 34$ [Taking $+4$ to the RHS gives -4, and taking x ($= +1x$) to the LHS gives $-1x$.]

$\therefore 5x - x = 34 - 4$ [Collecting like terms.]

$\therefore 4x = 30$

$\therefore x = \frac{30}{4}$ [Dividing by 4.]

$\therefore x = 7\frac{1}{2}$

4 $4y = 21 - 3y$

$\therefore 4y + 3y = 21$ [When solving equations, keep the equal signs underneath each other.]

$\therefore 7y = 21$

$\therefore y = \frac{21}{7}$

$\therefore y = 3$

Equations in Geometry

Equations can be used to solve problems in geometry.

Some Important Geometrical Facts

You need to know these facts so you can form equations:

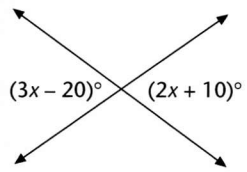
- Complementary angles add up to 90°.
- Supplementary angles add up to 180°.

- Angles on a straight line add up to 180°.
- Vertically opposite angles are equal.
- Angles at a point always add up to 360°.
- The angle sum of a triangle is 180°.
- The angle sum of a quadrilateral is 360°.
- The base angles of an isosceles triangle are equal.
- Any angle in an equilateral triangle is equal to 60°.
- The exterior angle of a triangle is equal to the sum of the two opposite interior angles.
- For a pair of parallel lines cut by a transversal:
  - Alternate angles are equal
  - Corresponding angles are equal
  - Co-interior angles are supplementary.

For Example

For the following diagrams, use a geometrical fact to form an equation. Solve the equation in each case to find x:

1



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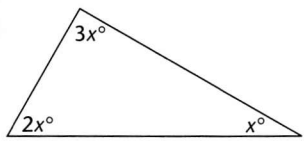

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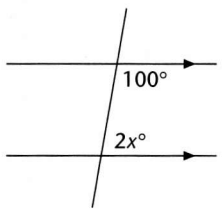

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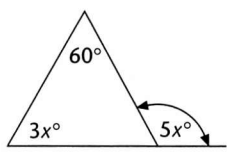

1 $3x - 20 = 2x + 10$ [Vertically opposite angles.]

$\therefore 3x - 2x = 10 + 20$

$\therefore x = 30$

[We do not use degrees in the equation as x is a number—units are not needed in either the question or the answer.]

2 $3x + 2x + x = 180$ [Angle sum of a triangle = 180.]

$\therefore 6x = 180$

$\therefore x = \frac{180}{6}$ [Collecting terms = $6x$.]

$x = 30$

3 $2x + 100 = 180$ [Co-interior angles are supplementary when the lines are parallel.]

$\therefore 2x = 180 - 100$

$\therefore 2x = 80$

$\therefore x = \frac{80}{2}$

$\therefore x = 40$

4 $5x = 3x + 60$ [Exterior angle of a triangle is equal to the sum of the opposite interior angles.]

$\therefore 5x - 3x = 60$

$\therefore 2x = 60$

$\therefore x = \frac{60}{2}$

$\therefore x = 30$

Word Problems Leading to Equations

Problems are often most easily solved by translating the facts given in words into an equation involving numbers and letters.

Procedure for Solving Word Problems

When solving word problems with one unknown, use the following procedure:

a Let the unknown quantity be represented by a pronumeral (say, x).

b Form an equation to represent the facts given in the problem.

c Solve the equation to find the value of the pronumeral.

d Translate the solution calculated into words to answer the question.

For Example

1 Five more than a number is equal to 17. Find the number.

1 Let the number be x. [step **a** of procedure]

Then $x + 5 = 17$ [step **b** of procedure]

$\therefore x = 17 - 5$ [step **c** of procedure: solve equation]

$\therefore x = 12$

$\therefore$ The number is 12. [step **d** of procedure]

For Example

1 If 7 is subtracted from 2 times a certain number, the result is 23. Find the number.

1 Let the number be x. [step **a** of procedure]

$\therefore 2x - 7 = 23$ [step **b** of procedure]

$2x = 23 + 7$

$2x = 30$

$x = 15$ [step **c** of procedure]

$\therefore$ The number is 15. [step **d** of procedure]

For Example

1 The sum of two consecutive integers is 33. (An integer is a whole number.) Find the two integers.

1 Let the consecutive integers be x and $x + 1$.

[step **a** of procedure]

[Note: Consecutive integers follow each other (e.g. 4, 5 ...). So if x is an integer, the next consecutive integer is $x + 1$.]

$\therefore x + x + 1 = 33$ [step **b** of procedure]

$2x + 1 = 33$

$2x = 33 - 1$

$2x = 32$

$x = \frac{32}{2}$

$x = 16$ [step **c** of procedure]

Therefore, the consecutive integers are 16 and 17.

[step **d** of procedure]