KEYWORDS

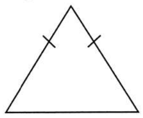
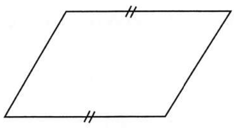
Acute
Adjacent
Alternate
Circle
Circumference
Co-interior
Complementary
Corresponding
Cross-section
Decagon
Dodecagon
Edges
Ellipse
Equilateral
Faces
Hendecagon
Heptagon
Hexagon
Isosceles
Kite
Nonagon
Obtuse
Octagon
Parallel
Parallelogram
Pentagon
Perpendicular
Platonic
Polygon
Polyhedron
Prism
Pyramid
Quadrilateral
Rectangle
Reflex
Revolution
Rhombus
Scalene
Skew
Square
Supplementary
Symmetry
Trapezium
Vertically opposite
Vertices

Notation and Conventions in Geometry

Symbol	Meaning
∟	A right angle (90°)
△	A triangle (e.g. ΔPQR)
^ or ∠	Angle (e.g. ∠ABC or $A\hat{B}C$)
|| or //	Parallel to (e.g. AB // PQ)
⊥	Perpendicular to (at 90° to)
°	Degrees (e.g. 27°)

Indicators

- To indicate that two intervals (sides) are equal, mark them in one of the following ways:


or

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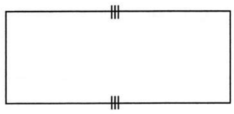
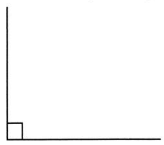
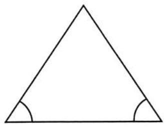


- To indicate that an angle is a right angle, mark it the following way:



- To indicate that two angles are equal, mark them in one of the following ways: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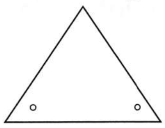
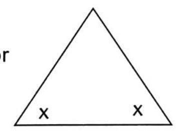
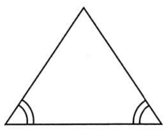
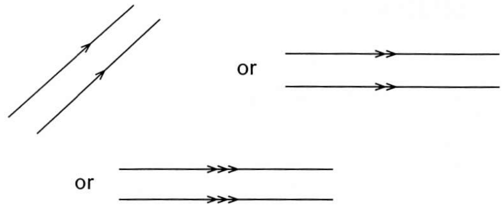
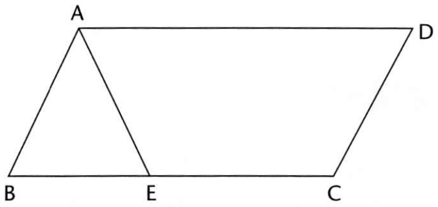

or

or


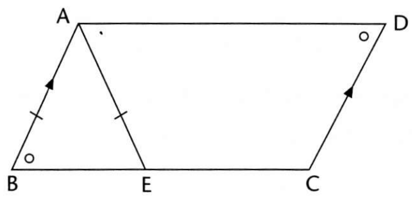
- To indicate that two lines are parallel, mark them as follows:





Draw the above figure in your book and mark on it the following information:

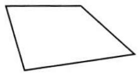
1. AB // DC
2. AB = AE
3. ∠ABE = ∠CDA

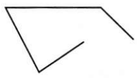


Polygons

A polygon is a closed figure with three or more straight sides.

Advise whether or not each shape is a polygon.

1  Polygon

2  Not a polygon (not closed)

3  Not a polygon (one side is not straight)

Naming Polygons

n stands for the number of sides of a polygon.

A polygon is named according to the number of sides it has:

Number of sides *n*	Name of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Hendecagon
12	Dodecagon
n	*n*-gon

Note: A polygon has the same number of angles as sides. For example, a hexagon has six angles and also six sides.

Regular Polygons

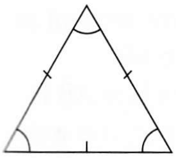
A polygon is **regular** if all its sides are equal in length and all its angles are equal.

Note:

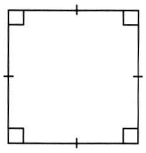
1 The regular triangle is an **equilateral triangle**.

2 The regular quadrilateral is a **square**.

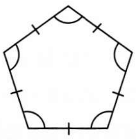
3 The regular five-sided polygon is called a **regular pentagon** etc.



Regular triangle
[equilateral triangle]

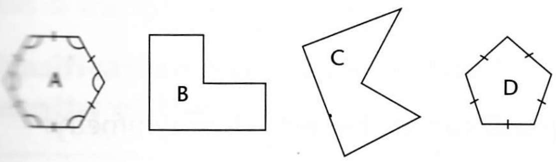


Regular
quadrilateral
[square]



Regular pentagon

For Example



Which of the figures A, B, C or D represent:

1 Hexagons?

2 Regular hexagons?

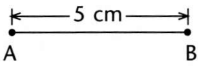
1 A and B are hexagons, since they are polygons with six sides.

2 A is a regular hexagon, since all six sides and angles are equal.

Naming Points, Intervals, Lines, Polygons

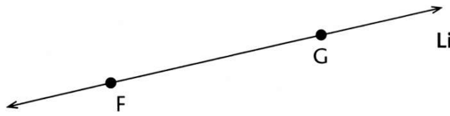
- A **point** is named by using capital letters; for example, B.

- An **interval** is named by using capital letters; for example:



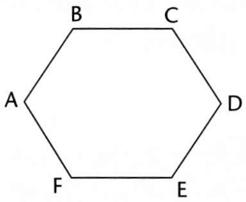
Interval AB = 5 cm

- A **line** is named by using any two points on it:

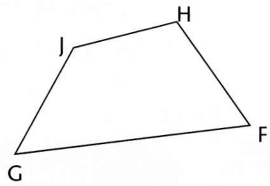


Line FG

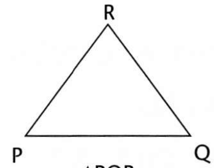
- **Polygons** are named by taking the vertices in order; for example:



Hexagon ABCDEF



Quadrilateral GFHJ



ΔPQR

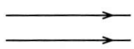
Note:

- △ stands for triangle.
- Three or more points on the one line are called **collinear points**.
- Three or more lines passing through the same point are called **concurrent lines**.

Lines

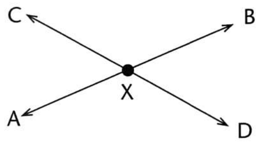
Two lines may be **parallel**, may **intersect** or may be **skew**.

- Parallel:



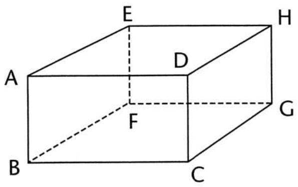
← These lines are parallel. (They are always the same distance apart.)

- Intersect at a point:

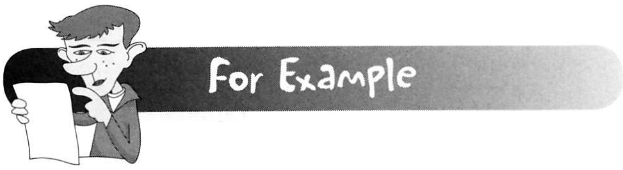


Line AB and line CD intersect at point X.

- Skew:

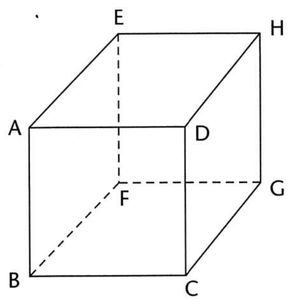


Line AB and line DH are skew. Skew lines do not intersect and are not parallel.



Name the lines in the figure that are:

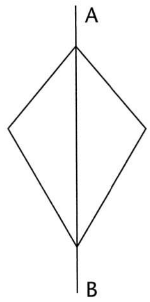
1. Parallel to AD
2. Perpendicular to AD
3. Skew to AD.



1. The lines parallel to AD are BC, EH and FG.
2. The lines perpendicular to AD are AB, DC, AE and DH.
3. The lines skew to AD are BF, EF, CG and GH.

Symmetry

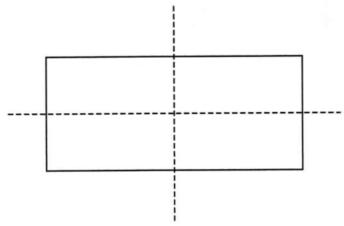
A figure has **line symmetry** if one side of the figure is a mirror image of the other about a line.



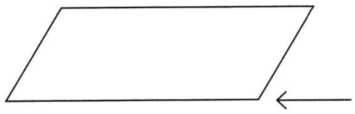
This figure is symmetrical about line AB. The line AB is called the **axis of symmetry**.

Some figures have more than one axis of symmetry and some have no axis of symmetry (i.e. they are not symmetrical).

A rectangle, for example, has two axes of symmetry:



A parallelogram has no axis of symmetry:

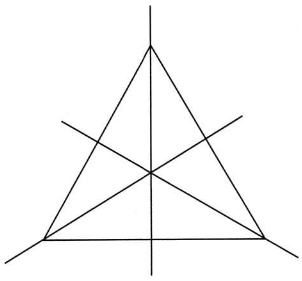


It is not symmetrical

Order of Symmetry

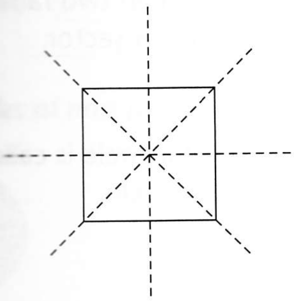
The **order of symmetry** is the number of axes of symmetry of a figure.

An equilateral triangle has three axes of symmetry. This figure is said to have line symmetry of order three:



For Example

1 Draw a square and mark on it all axes of symmetry. What is the order of symmetry of a square?

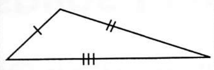
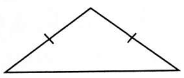
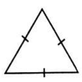


The order of symmetry = 4.

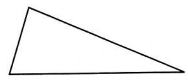
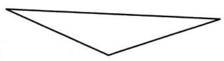
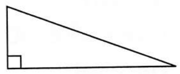
Classification of Triangles

A triangle is a polygon with three sides, and can be classified according to the lengths of its sides or the sizes of its angles.

Classification According to the Lengths of the Sides

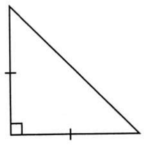
Name	Diagram	Properties
Scalene		All sides are different lengths
Isosceles		Two sides are equal in length
Equilateral		All sides are equal in length

Classification According to the Size of the Angles

Name	Diagram	Properties
Acute-angled		All angles are acute
Obtuse-angled		One angle is obtuse
Right-angled		One angle is a right angle

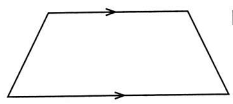
Note: 'Acute' and 'obtuse' are defined in the section 'Angle Terminology'.

Note: A triangle can have the properties of two types of triangles. For example, it can be both isosceles and right-angled, as in this example:



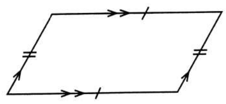
Classification of Quadrilaterals

Trapezium



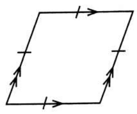
- A pair of sides are parallel

Parallelogram



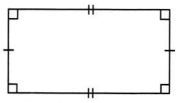
- Opposite sides are equal in length
- Opposite sides are parallel

Rhombus
(diamond shape)



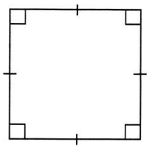
- All sides are equal in length
- Opposite sides are parallel

Rectangle



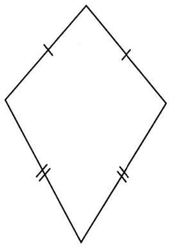
- Opposite sides are equal in length
- Opposite sides are parallel
- All angles are right angles

Square



- All sides are equal in length
- Opposite sides are parallel
- All angles are right angles

Kite

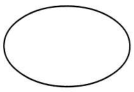


- Two pairs of adjacent sides are equal

See page 182 for more work on quadrilaterals.

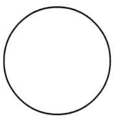
Other Plane Shapes

Ellipse (oval)

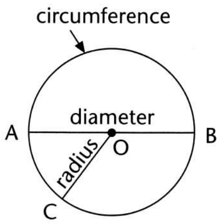


[Note the egg shape.]

Circle



Parts of a Circle

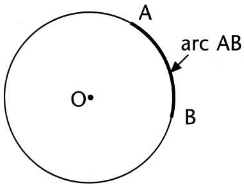


The centre of a circle is usually named O.

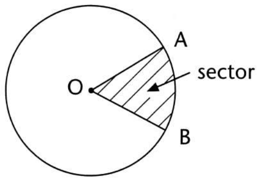
The circumference of a circle is its perimeter.

AB is the diameter.

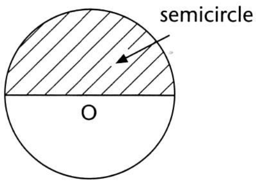
OC is the radius. The radius is half the length of the diameter.



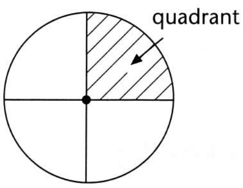
The length AB (on the circumference) is called an arc.



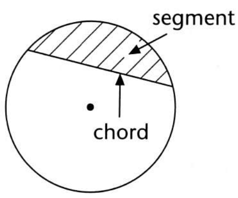
The area contained between two radii is called a sector.



Half a circle is called a semicircle.



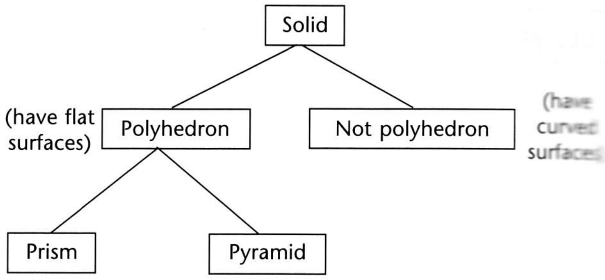
A quarter of a circle is called a quadrant.



A chord joins any two points on the circumference.
A segment is the area contained between a chord and the edge of the circle.

Classification of Solids

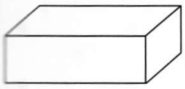
An object that occupies space is called a solid; for example, a textbook, a ball, a cone. Solids are classified in the following way:



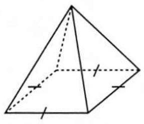
Definitions

A polyhedron is a solid made of plane surfaces with straight edges (flat surfaces).

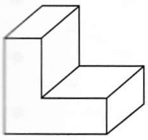
Examples of polyhedra

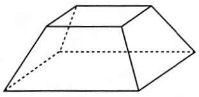


Rectangular prism



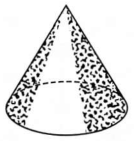
Square pyramid



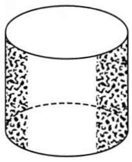


Examples of non-polyhedra

Sphere



Cone

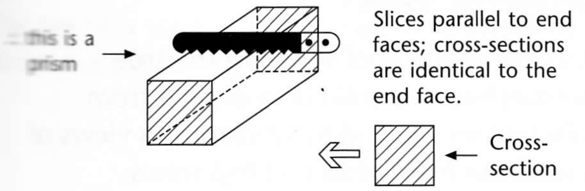


Cylinder

Note: Polyhedra is the plural for polyhedron.

Prisms

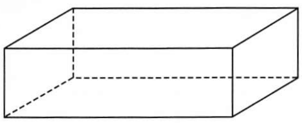
A prism is a solid that has a pair of parallel faces and, if 'sliced' parallel to these faces, the cross-sections are identical to the end faces:



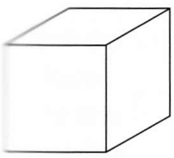
(Remember: You can sit comfortably on a prism.)

Note: A prism is named according to the shape of its cross-section.

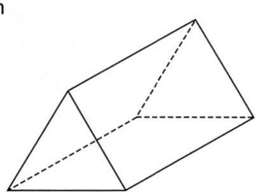
Some familiar prisms



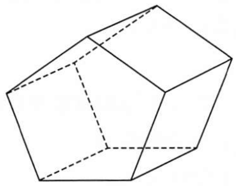
Rectangular prism



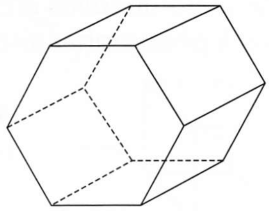
Cube



Triangular prism



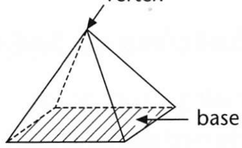
Pentagonal prism



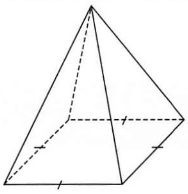
Hexagonal prism

Pyramids

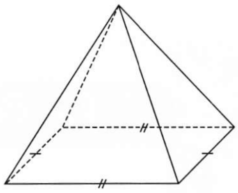
A pyramid is named according to the shape of its base.



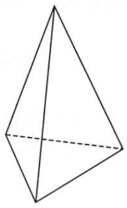
Some familiar pyramids



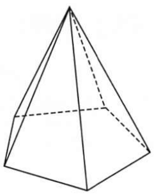
Square pyramid



Rectangular pyramid



Triangular pyramid



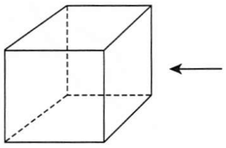
Pentagonal pyramid

Platonic Solids

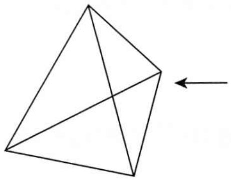
Platonic solids are polyhedra whose faces are all congruent (identical) regular polygons. The most common example is a hexahedron (cube) whose faces are six identical squares.

Another example of a platonic solid is a regular tetrahedron, which is a triangular pyramid whose faces are four identical equilateral triangles.

The platonic solids are named after the ancient Greek philosopher and teacher Plato:



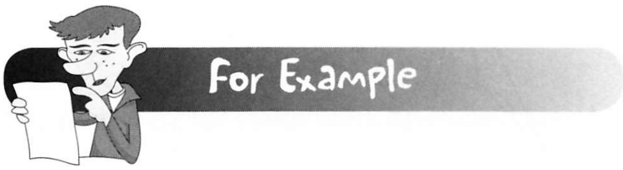
A cube is a platonic solid. It is also called a hexahedron.



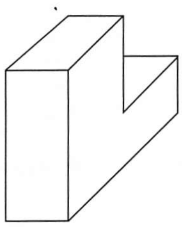
A triangular pyramid with four identical equilateral triangles as faces is called a tetrahedron.

Isometric Sketches of Solids

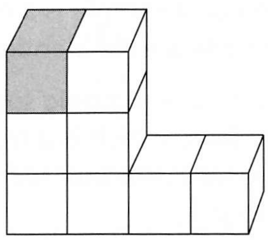
Solids are difficult to represent on paper, due to their three-dimensional qualities. Sketching them on isometric dot paper allows the solid to be drawn showing its true perspective.



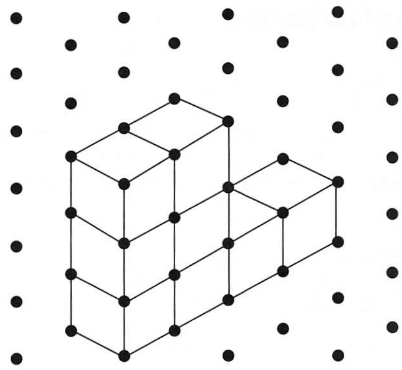
1 Draw the following solid on isometric dot paper:



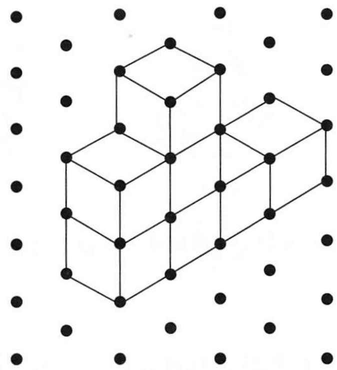
2 Draw a new solid by removing the shaded solid:



1

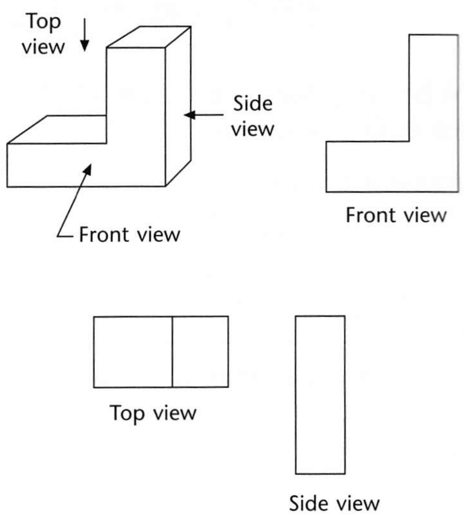


2



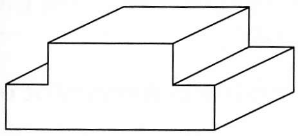
Sketching Solids from Different Views

Another method of showing the true perspective of a solid is to draw it from different views. The most common views of a solid are front, side and top views:



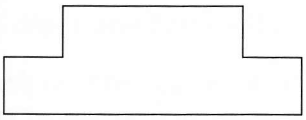
For Example

1 Consider the following solid:

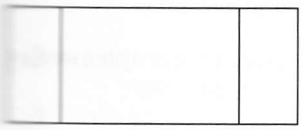


Draw the front, side and top views of the solid.

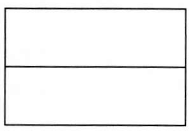
1



Front view



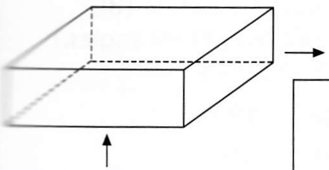
Top view



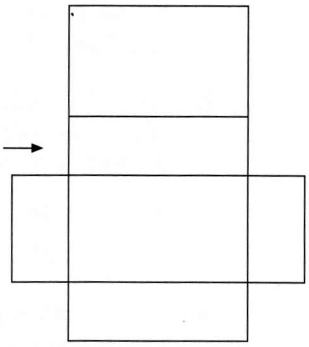
Side view

Nets of Solids

The net of a solid is a plan that, when folded, forms the solid:



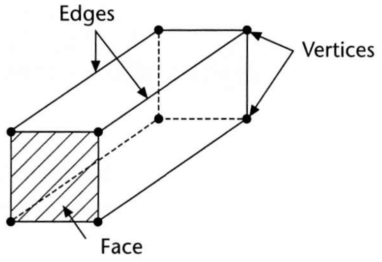
This is a rectangular prism made of cardboard



Net of a rectangular prism

Parts of Polyhedra

- Each surface of a polyhedron is called a **face**.
- The line in which two faces meet is called an **edge**.
- The point where edges meet is called a **vertex** (corner point).



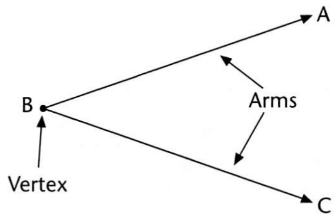
Note: Vertices is the plural for vertex.

Angle Terminology

Naming Angles

When naming angles, we start from arm → vertex → arm.

For example, the angle in the diagram below is ∠ABC or ∠CBA:



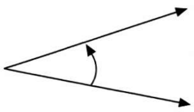
Notation

For the angle in the above diagram, we can use any of the following notations:

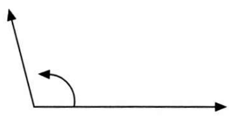
- ∠ABC
- $A\hat{B}C$
- $\hat{B}$ or ∠B

Types of Angles

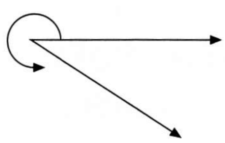
We name angles as follows:



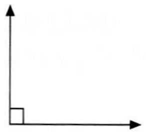
Acute angle—measures between 0° and 90°.



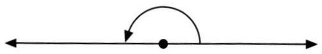
Obtuse angle—measures between 90° and 180°.



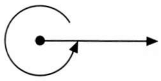
Reflex angle—measures between 180° and 360°.



Right angle—measures 90°.



Straight angle—measures 180°.

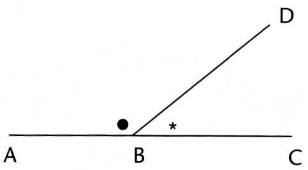


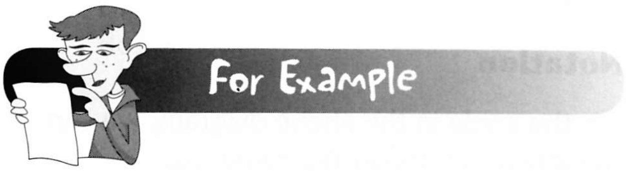
Revolution—measures 360°.

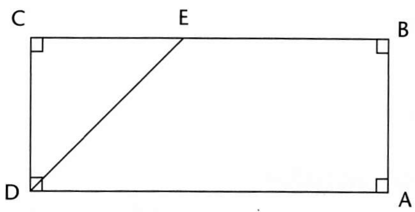
Complementary angles—two angles that add up to 90°.

Supplementary angles—two angles that add up to 180°.

Adjacent angles—two angles are adjacent if they have a common arm and common vertex. In the diagram, ∠ABD is adjacent to ∠DBC. (BD is the common arm, and B is the common vertex.)







In the figure, ABCD is a rectangle.

1 From the diagram, name:

- **a** An acute angle
- **b** An obtuse angle
- **c** A right angle
- **d** A straight angle
- **e** A pair of complementary angles
- **f** A pair of supplementary angles.

2 From the diagram, answer these questions:

- **a** If ∠CDE = 40°, find the size of ∠EDA.
- **b** If ∠BED = 130°, find the size of ∠DEC.

3 **a** Find the complement of 70°.

- **b** What is the supplement of 70°?

1 **a** ∠CDE is an acute angle (also ∠EDA, ∠CED).

- **b** ∠BED is an obtuse angle.
- **c** There are four right angles in the diagram, $\hat{A}$, $\hat{B}$, $\hat{C}$ and ∠CDA.
- **d** ∠BEC is a straight angle.
- **e** ∠CDE and ∠EDA are complementary since ∠CDE + ∠EDA = 90°.
- **f** ∠BED and ∠DEC are adjacent supplementary angles since ∠BED + ∠DEC = 180°.

2 **a** From 1e, ∠CDE + ∠EDA = 90°
(They are complementary angles.)
If ∠CDE = 40°, then 40° + ∠EDA = 90°
∴ ∠EDA = 50°

b From 1f, ∠BED + ∠DEC = 180°
(They are supplementary angles.)
If ∠BED = 130°, then

130° + ∠DEC = 180°

∴ ∠DEC = 50°

3 **a** To find the **complement** of 70° we find an angle that, when added to 70°, makes 90°. Therefore, the complement of 70° = 20°.

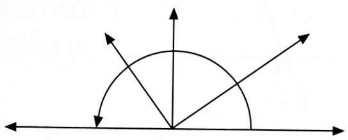
[The complement of $x° = (90 - x)°$]

b To find the **supplement** of 70° we find an angle that, when added to 70°, makes 180°. Therefore, the supplement of 70° = 110°.

[The supplement of $x° = (180 - x)°$]

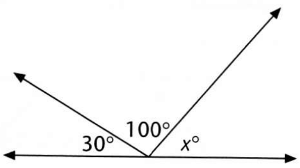
Further Angles

Angles on a straight line



Angles on a straight line add up to 180°.

For Example



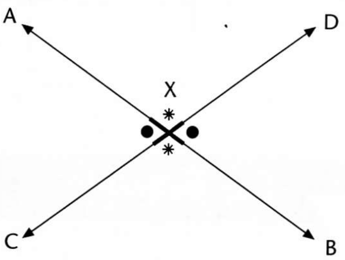
1 Find x.

1 $x + 100 + 30 = 180$

(angles on a straight line)

$\therefore\ x = 50$

Vertically opposite angles

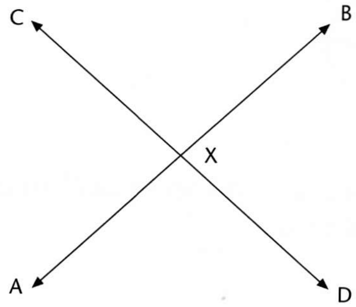


[Lines AB and CD intersect at X.]

When two lines intersect, two pairs of vertically opposite angles are formed. Vertically opposite angles are equal.

$\angle$CXA and $\angle$DXB are vertically opposite. Also, $\angle$BXC and $\angle$AXD are vertically opposite.

For Example



[AB intersects CD at X]

1 Name the angle vertically opposite to $\angle$BXD.

2 If $\angle$AXC = 112°, find the size of $\angle$BXD.

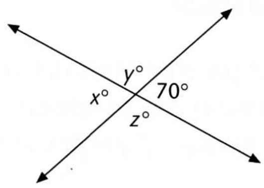
3 Does $\angle$CXB = $\angle$DXA? Why?

1 The angle vertically opposite to $\angle$BXD is $\angle$AXC.

2 $\angle$BXD = $\angle$AXC (because they are vertically opposite) $\therefore$ $\angle$BXD = 112°

3 $\angle$CXB = $\angle$DXA because they are vertically opposite angles (and vertically opposite angles are equal).

For Example



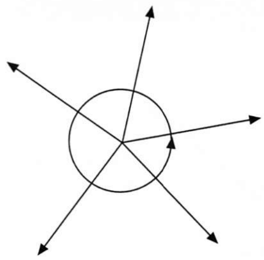
1 Find the values of x, y and z.

1 $x = 70$ (vertically opposite angles)

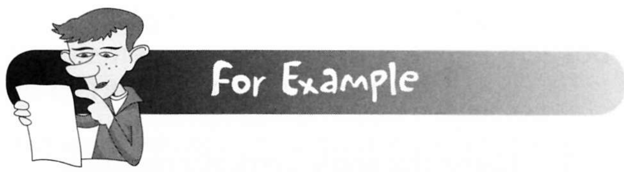
$y + 70 = 180$ (supplementary angles)

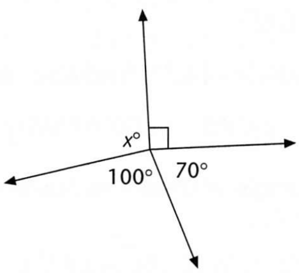
$\therefore\ y = 110,\ z = 110$

Angles at a point



Angles at a point add up to 360° (make up one revolution).





1 Find x.

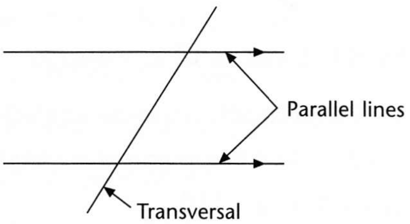
1 $x + 100 + 70 + 90 = 360$ (angles at a point)

$\therefore\ x + 260 = 360$

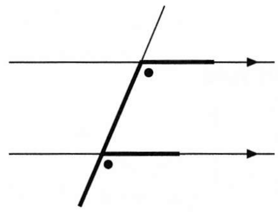
$x = 100$

Parallel Lines

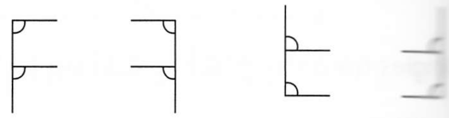
When a pair of parallel lines is cut by a transversal, three types of special angles are formed: F angles, Z angles and C angles.



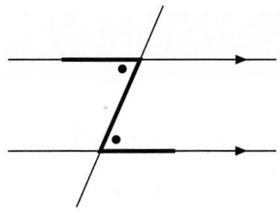
1



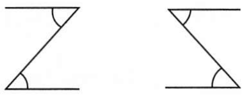
F angles—corresponding angles are equal.



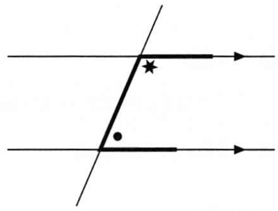
2



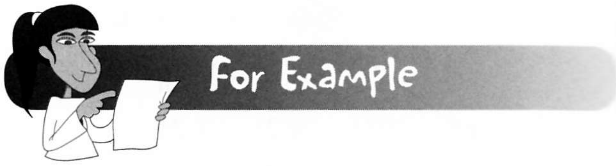
Z angles—alternate angles are equal.

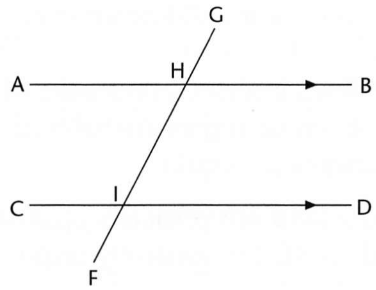


3



C angles—co-interior angles are supplementary; that is, they add up to 180°.





1 In the figure, AB // CD and AB and CD are cut by the transversal FG:

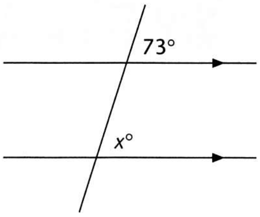
a Which angle is corresponding to $\angle$HID?

b Which angle is alternate to ∠HID?

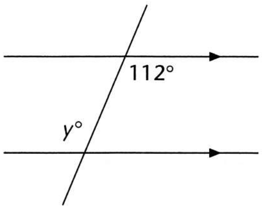
c Which angle is co-interior to ∠HID?

d If ∠HID = 70°, find the size of ∠BHI.

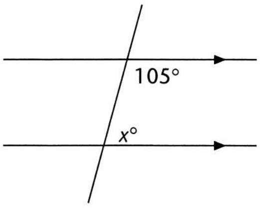
2 a Find x.



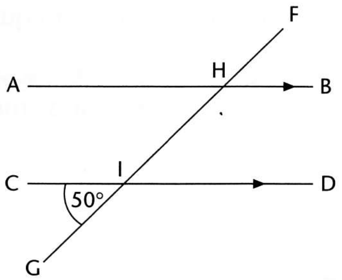
b Find y.



c Find x.

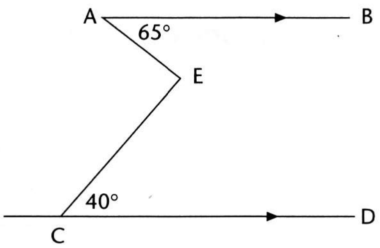


3 a



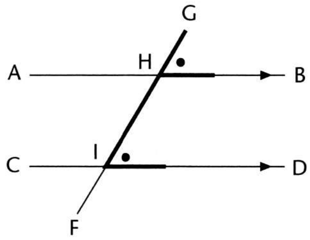
AB // CD and ∠GIC = 50°.
Find the size of ∠BHI.

b



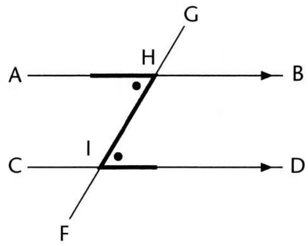
AB // CD. Find the obtuse angle ∠CEA.

1 a The angle corresponding to ∠HID is ∠GHB:

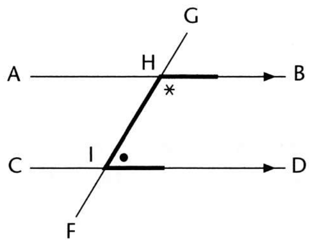


Remember that corresponding angles or alternate angles are only equal when the lines are parallel.

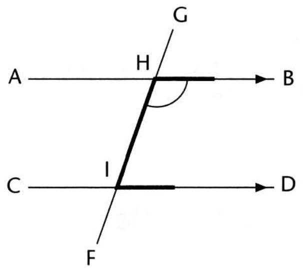
b The angle alternate to ∠HID is ∠IHA:



c The angle co-interior to ∠HID is ∠BHI:



d



∠BHI + ∠HID = 180°
(co-interior angles, AB // CD)
∴ ∠BHI + 70° = 180°
∠BHI = 110°

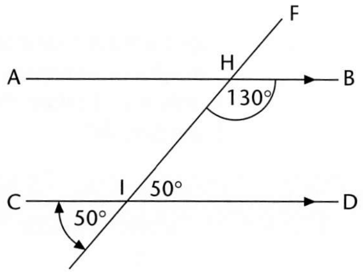
Remember, co-interior angles are only supplementary when the lines are parallel.

2 **a** $x = 73$ (corresponding angles in parallel lines)

b $y = 112$
(alternate angles in parallel lines)

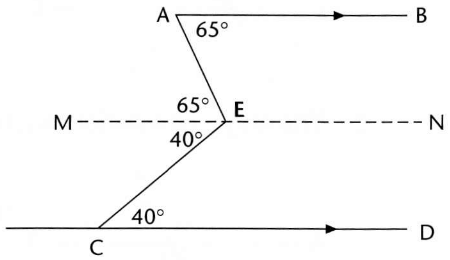
c $x + 105 = 180$
(co-interior angles in parallel lines)
$\therefore x = 75$

3 **a**



$\angle HID = 50°$ (vertically opposite to $\angle GIC$)
$\angle BHI = 130°$ (co-interior to $\angle HID$, and AB // CD)

b



Construct a line MN through E that is parallel to AB. Since MN // AB and AB // CD, then MN // CD.

$\therefore \angle MEA = 65°$ (alternate to $\angle BAE$, AB // MN)

Also, $\angle CEM = 40°$ (alternate to $\angle ECD$, CD // MN)

$\therefore \angle CEA = (65 + 40)° = 105°$

Conditions for Parallel Lines

Two straight lines are parallel if a transversal makes with them:

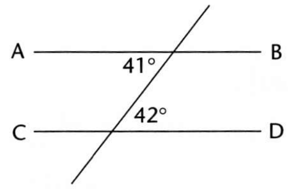
i A pair of equal corresponding angles, or

ii A pair of equal alternate angles, or

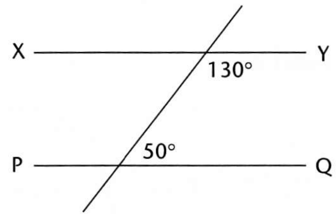
iii A pair of supplementary co-interior angles.

Also, two straight lines are parallel if they are both parallel to a third line.

1 Is AB // CD?



2 Are the lines XY and PQ parallel?



1 AB is not parallel to CD, because the pair of alternate angles are not equal.

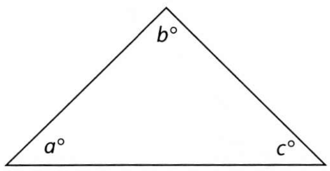
2 XY and PQ are parallel, because the pair of co-interior angles are supplementary.

That is, $130° + 50° = 180°$.

Triangles

Properties of Triangles

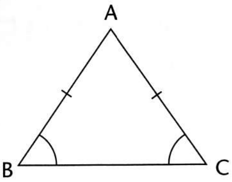
Angle sum of a triangle



The angle sum of any triangle is 180°.

$[a + b + c = 180]$

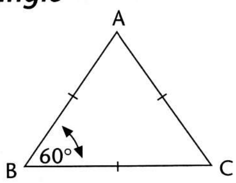
Isosceles triangle



The base angles of an isosceles triangle are equal.

[$\angle ABC = \angle BCA$]

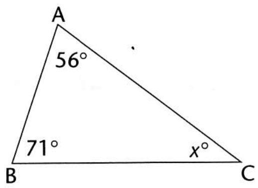
Equilateral triangle



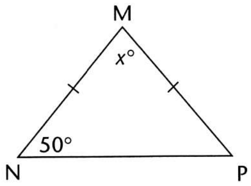
Each angle in an equilateral triangle is 60°.

[$\angle A = \angle B = \angle C = 60°$]

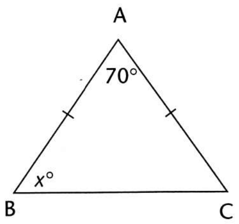
1 Find x.



2 Find x.



3 Find x.



1 $x + 71 + 56 = 180$ (angle sum of $\triangle ABC$)

$x + 127 = 180$

$x = 53$

2 $\angle NPM = 50°$

(base angles of isosceles $\triangle MNP$)

$\therefore\ x + 50 + 50 = 180$

(angle sum of $\triangle MNP$)

$\therefore\ x = 80$

3 $\angle ABC = \angle BCA$

(base angles of isosceles $\triangle ABC$)

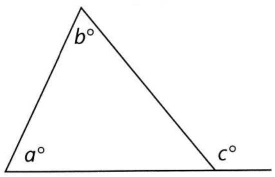
$x + x + 70 = 180$ (angle sum of $\triangle ABC$)

$2x = 110$

$\therefore\ x = 55$

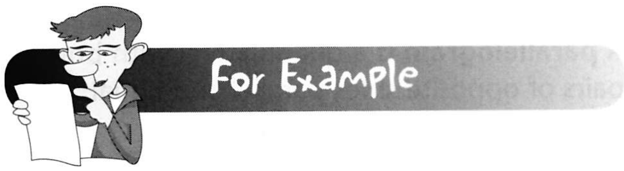
Exterior Angle of a Triangle (exterior = outside)

The exterior angle of any triangle equals the sum of the two opposite interior angles:

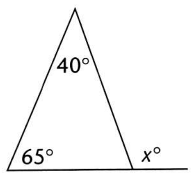


[$c = a + b$]

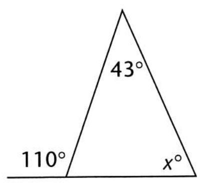
Note: The opposite interior angles are the two that are **not** adjacent to the exterior angle.



1 Find x.



2 Find x.

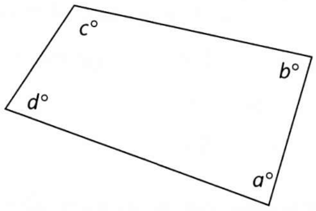


1 $x = 65 + 40$ (exterior $\angle$ of $\triangle$ result)
$\therefore x = 105$

2 $x + 43 = 110$ (exterior $\angle$ of $\triangle$ result)
$\therefore x = 110 - 43$
$\therefore x = 67$

Quadrilaterals

Angle Sum of Quadrilaterals



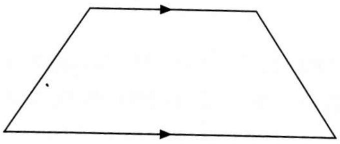
The angle sum of any quadrilateral is 360°.

That is: $\boxed{a + b + c + d = 360}$

Properties of Special Quadrilaterals

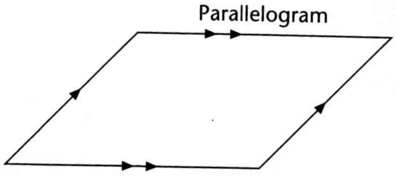
A **trapezium** is a quadrilateral with one pair of opposite sides parallel:

Trapezium



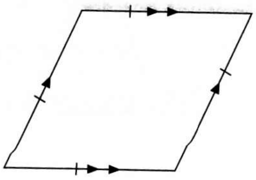
A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel and equal:

Parallelogram



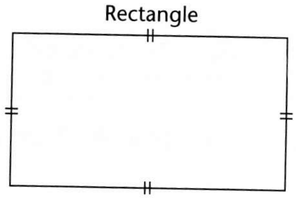
A **rhombus** is a quadrilateral with both pairs of opposite sides parallel and all sides equal:

Rhombus



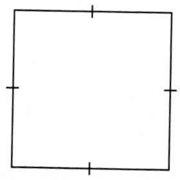
A **rectangle** is a quadrilateral with both pairs of opposite sides parallel and equal, and all four angles are right angles:

Rectangle



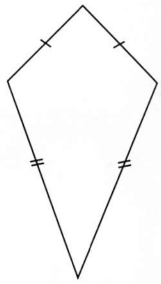
A **square** is a quadrilateral with all four sides equal and all four angles are right angles:

Square

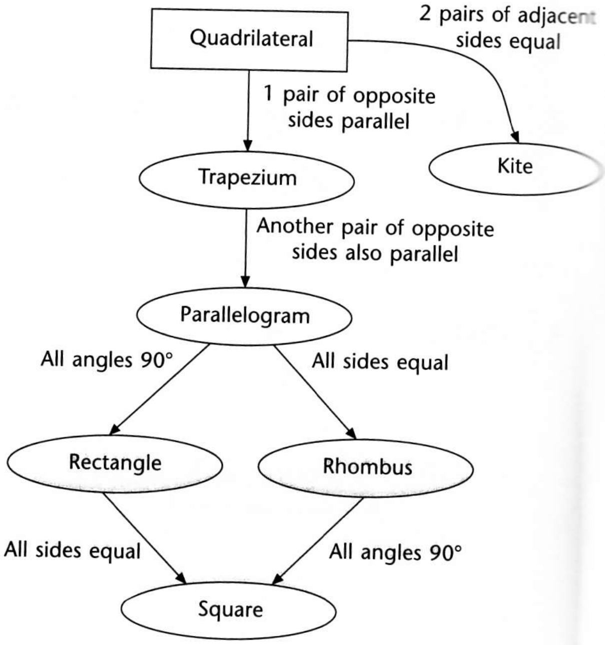


A **kite** is a quadrilateral with two pairs of adjacent sides equal:

Kite

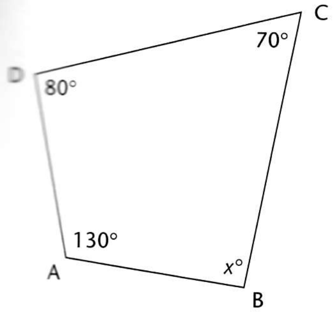


Family of Quadrilaterals



For Example

1 Find x, giving reasons for your answer:



1 $x + 130 + 80 + 70 = 360$

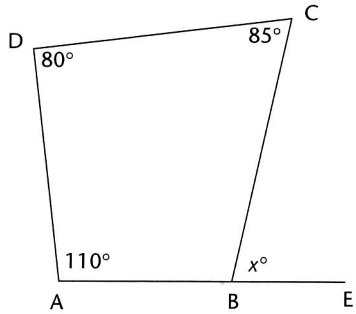
(Angle sum of quadrilateral ABCD.)

$\therefore\ x + 280 = 360$

$x = 80$

For Example

1 Find x, giving reasons for your answer:



1 $\angle ABC + 85° + 80° + 110° = 360°$

(Angle sum of quadrilateral ABCD.)

$\angle ABC + 275° = 360°$

$\angle ABC = 85°$

$\therefore\ x + 85 = 180$

($\angle CBE$ is supplementary to $\angle ABC$.)

$\therefore\ x = 95$