Introduction to Algebra and Equations

Algebra is a part of mathematics which generalises on quantities, so instead of using numbers, symbols (mostly letters) are used to represent values.

Example:

The university is buying cars to replace aging vehicles. They decide to buy 5 Astras, 7 Barinas, 10 Commodores and 4 Statesmans.

Let the cost of an Astra be a, Barina b, Commodore c, and Statesman s, then the total cost of all the vehicles can be represented by the expression: (Note $5 \times a$ can be written as $5a$ if a variable is used.)

$$\begin{aligned}&5 \times a + 7 \times b + 10 \times c + 4 \times s \\ &= 5a + 7b + 10c + 4s\end{aligned}$$

The university will obtain quotes from different companies which will result in different total costs, however, the algebraic expression above will always be true. As each dealer supplies quotes, the university can substitute the values into the expression and calculate the total cost. If one dealer quotes an Astra at \$25000, a Barina at \$16000, a Commodore at \$35000 and a Statesman at \$55000, then the total cost will be:

$$\begin{aligned}&5a + 7b + 10c + 4s \\ &= 5 \text{ x } 25000 + 7 \text{ x } 16000 + 10 \text{ x } 35000 + 4 \text{ x } 55000 \\ &= \$807\,000\end{aligned}$$

This process can be repeated for different quotes.

Example:

A father and daughter have the same birth date but were born 25 years apart. An equation describing this situation is:

F = D + 25 where F is the father's age and D is the daughter's age.

This equation states that the Father's age is equal to the Daughter's age plus 25. When the daughter is 10, the father's age, F = 10 + 25 = 35.

Language of Algebra

Variables – variables are letters that are used to represent an unknown quantity. In the car example above, the price of the Astra is represented by the variable a, because the price is unknown and can vary depending on the supplier. Another name for variable is pronumeral , indicating that this letter is representing a number.

In the second example, the variables F, D were used. The equation means that there is a relationship between the two variables. Often, letters are chosen that reflect the quantity being represented; h – height of the tree, t – time taken to…etc.

Sometimes variables are used with a subscript, for example, d_1. There are instances when formulas contain two distance measurements, the variable d is used for both and a subscript is used to distinguish each distance. For example d_0 could mean the initial distance and d_t could mean the distance after time t.

Terms – $2a,\ ef,\ 8,\ a^2b,\ \frac{3}{x}$ are all terms. Terms are variables, numbers or a combination of variables and numbers involving multiplication or division. Some of the terms listed are variable terms ($2a,\ ef,\ a^2b,\ \frac{3}{x}$) because they contain a variable. The term 8 is called a constant term because 8 is constant (it doesn't change).

The term $2a$ means $2\times a$. The multiplication sign is mostly omitted and it is conventional to write the number before the variable. When the term contains more than one variable, the variables are written in alphabetical order. The term dc3b should be written as 3bcd following convention. In the term $2a$, the 2 is called the **coefficient** of a.

An **expression** is a collection of terms being added or subtracted.

For example: Consider the expression $3a+4b-c+7$

This expression contains 4 terms
There are 3 variable terms ($3a, 4b, -c$) and 1 constant term (7).
The coefficient of the a is 3
The coefficient of the b is 4
The coefficient of the c is $^{-}1$

Note: when a variable is written with no coefficient, the implied coefficient is 1.

So f is the same $1f$, abc is the same as $1abc$, ^{-}h is the same as $^{-}1h$.

Revision of exponents

The **power** x^n can be written in expanded form as: $x^n = x \times x \times x \times x \times x .... \times x$ $\left[\text{for } n \text{ factors}\right]$

The **power** x^n consists of a **base** x and an **exponent** (or index) n.

Base $\rightarrow$ x^n $\leftarrow$ Exponent (or index)

'The number of times it is repeated.'

'The number being repeated.'

The table below contains a summary of exponent rules.

Rule	Examples
$x^n = x \times x \times x \times x \times x \times x......x$ for n terms	$3^4 = 3 \times 3 \times 3 \times 3 = 81$ $x^7 = x \times x \times x \times x \times x \times x \times x$
$x^m \times x^n = x^{m+n}$	$3^4 \times 3^5 = 3^{4+5} = 3^9$ $x^3 \times x^2 = x^{3+2} = x^5$
$x^m \div x^n = \dfrac{x^m}{x^n} = x^{m-n}$	$\dfrac{7^8}{7^5} = 7^{8-5} = 7^3$ $x^5 \div x^3 = x^{5-3} = x^2$
$\left(x^m\right)^n = x^{m \times n}$	$\left(2^4\right)^3 = 2^{4\times3} = 2^{12}$ $\left(x^4\right)^{\frac{1}{2}} = x^{4\times\frac{1}{2}} = x^2$
$a^{-n} = \dfrac{1}{a^n}$ **or** $\dfrac{1}{a^{-n}} = a^n$	$3^{-5} = \dfrac{1}{3^5}$ $\dfrac{1}{x^{-3}} = x^3$

When working in Algebra, there are times when **Order of Operations** is important.

When solving an expression containing different operations, use this order.

1. Evaluate brackets or other grouping symbols first. If there are nested brackets (brackets within brackets) work from the inner set to the outer set.

2. Evaluate any powers or roots. This is covered in the indices module. An example of a simple power is $3^2 = 3 \times 3 = 9$. An example of a root is $\sqrt{16} = 4$ because $4 \times 4 = 4^2 = 16$.

3. Multiplication and division. These operations are equal in priority and an expression containing both operations should be solved working left to right.

4. Addition and subtraction. These operations are equal in priority and an expression containing both operations should be solved working left to right.

Video 'Language of Algebra'

Module contents

Introduction

- Substitution into expressions, equations and formulae
- Adding and subtracting like terms
- Multiplying and dividing terms
- Expanding brackets
- Factorising

Outcomes

- To substitute into Algebraic expressions, Equations and Formulae
- To identify like terms and perform addition or subtraction on these
- To multiply and divide algebraic terms
- To expand brackets using the Distributive Law
- To factorise algebraic expressions using common factors
- To rearrange equations and formulae.

Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module. Check your answers from the answer section at the end of the module.

1. If $a=7$, $b=3$, $c=0.4$ and $d={}^{-}6$, calculate the value of:

   (a) $ac-b$ (b) $d^2-\frac{ab}{c}$

2. (a) Simplify $3ab + 7c + ab - 5c$.

   (b) Simplify $2ab^2-3a^2b-b^2a$

3. (a) Simplify $3ab\times 6bc\times {}^{-}2d^2$

   (b) Simplify $4abc \div 8abd$

4. Expand and collect like terms $3x(a-5)+4x$

5. Factorise the expression $5ab-15ac$

6. (a) Solve the equation $\frac{4c-7}{7}=3$ (b) Make B the subject in $W=\frac{B^2}{2\mu}$

Topic 1: Substitution into expressions, equations and formulae

The term ‘substitution’ means that the value of the variable is known and the variable will be replaced by the value given for each variable.

Examples

Evaluate (find the value of) the expression $4a+5b-c$ when $a=5,\ b=2$ and $c={}^{-}3$

$$\begin{aligned}&4a+5b-c\\&=4\times5+5\times2-{}^{-}3\\&=20+10+3 \qquad (-{}^{-}\rightarrow+)\\&=33\end{aligned}$$

Evaluate b^2-4ac when $a=1, b=8$ and $c=-7$

$$\begin{aligned}&b^2-4ac\\&=8^2-4\times1\times{}^{-}7\\&=64+28\\&=92\end{aligned}$$

Evaluate $\frac{a}{b}-\frac{1}{2}$ when $a=4$ and $b={}^{-}5$

$$\begin{aligned}&\frac{a}{b}-\frac{1}{2}\\&=\frac{4}{{}^{-}5}-\frac{1}{2} \qquad \left(\frac{4}{{}^{-}5}=\frac{{}^{-}4}{5}\right)\\&=\frac{{}^{-}4}{5}-\frac{1}{2}\\&=\frac{{}^{-}8}{10}-\frac{5}{10}\\&=\frac{{}^{-}13}{10}\ \text{or}\ {}^{-}1\frac{3}{10}\end{aligned}$$

Evaluate $\sqrt{p^3+q^2-r}$ when $p=3.1, q=1.9$ and $r=5.5$

$$\begin{aligned}
&\sqrt{p^3+q^2-r} \\
&=\sqrt{(3.1)^3+(1.9)^2-5.5} \\
&=\sqrt{29.791+3.61-5.5} \\
&=\sqrt{27.901} \\
&\simeq 5.282 \text{ (to 3 d.p.)}
\end{aligned}$$

In the equation $y=3x+4$, find the value of y when $x=5$

$$\begin{aligned}
y&=3x+4 \\
y&=3\times5+4 \\
y&=15+4 \\
y&=19
\end{aligned}$$

In the formula $V=\dfrac{2d_1}{d_1-d_2}$, find the value of V when $d_1=4500$ and $d_2=1250$

$$\begin{aligned}
V&=\frac{2d_1}{d_1-d_2} \\
V&=\frac{2\times4500}{4500-1250} \\
V&=\frac{9000}{3250} \\
V&=2.77 \text{ (to 2 d.p.)}
\end{aligned}$$

Video 'Substitution'

Activity

1. Evaluate the following expressions when $m = 5$ and $n = 12$
   (a) $m + n$
   (b) $2m - n$
   (c) $m^2 - 5m + 6$
   (d) $(n - m)^2 + 2n - m$

2. Evaluate the following expressions when $a = 2$, $b = {}^{-}3$ and $c = \frac{1}{4}$
   (a) $2a - 3b + 4c$
   (b) $a \div c$
   (c) $ab^2 - a^2c$
   (d) $\frac{b}{a} + c$

3. The speed (s) of a car is given by the equation $s = \frac{d}{t}$, where d is the distance covered by the car in kilometres and t is the time taken to cover that distance in hours.
   Calculate the speed of the car if it covers 250km in 3 hours.

4. The Potential Energy (P) of a moving object is given by the formula $P = mgh$ where m is the mass of the object in kilograms, g is the acceleration due to gravity in $m / \sec^2$ and h is the height above the ground in metres.
   Find the potential energy of 550kg object when it is 10m above the ground, given the g=9.8 $m / \sec^2$

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Topic 2: Adding and subtracting like terms

In order to add or subtract algebraic terms, the terms must be like terms, that is, the terms must have exactly the same variable(s).

Examples

$2a$ and $7a$ are like terms because the variable is the same in both terms.

Because the terms are like, they can be added, ie: $2a + 7a = 9a$ This can be also thought of as ‘ 2 lots of a mystery number plus 7 lots of a mystery number gives 9 lots of a mystery number’.

$3x$ and $3y$ are **not** like terms because the variable is different in each term. So $3x$ and $3y$ cannot be added or subtracted.

$3ab$ and $5ab$ are like terms. So $3ab + 5ab = 8ab$ or $3ab - 5ab = {}^{-}2ab$

$4ab$ and $9ba$ are also like terms because $ab\ (a \times b)$ is the same as $ba\ (b \times a)$. So $4ab + 9ba = 4ab + 9ab = 13ab$.

$5ab$ and $3a$ are **not** like terms because the variables must be the same. So $5ab$ and $3a$ cannot be added or subtracted.

a^2 and $4a^2$ are like terms, where $3a^2$ and $3a$ are **not** like terms.

$2abc$ and $4cab$ are like terms, where $5abc$ and $3ac$ are **not** like terms.

Variable terms and constant terms are not like terms.

Examples

Simplify these expressions if possible

	Question	Solution
1.	$4a+11a$	$4a+11a$ $=15a$
2.	$4ab-6a$	Cannot be done
3.	$3t+7t-4t$	$3t+7t-4t$ $=10t-4t$ $=6t$
4.	$3gh+6hg-15gh$	$3gh+6hg-15gh$ $=9gh-15gh$ $={}^{-}6gh$
5.	$3a+6b-a+3b$ When re-ordering terms move the term and the sign in front of it.	$3a+6b-a+3b$ $=3a-1a+6b+3b$ $=2a+9b$
6.	$6fg-4f+9-fg+2f-7$	$6fg-4f+9-fg+2f-7$ $=6fg-1fg-4f+2f+9-7$ $=5fg-2f+2$

Video ‘Adding and subtracting like terms’

Activity

1. Simplify these expressions by collecting like terms if possible

   (a) $14p - 7p$

   (b) $^{-}j - 3j$

   (c) $4ab + 3a - ab$

   (d) $3x + y + 7x - 6y$

   (e) $4x^2 + 4x - 2 + x^2 - 8x + 5$

   (f) $4a + 1 - 9a + 11$

   (g) $^{-}q + 3q - 7 + 4q$

   (h) $4de + 9ed - de + 4e$

   (i) $9 - 2s + 2 + 2s$

   (j) $7 - a + 5a^2 - a^3 - 2 + 2a - 3a^2 + a^3$

2. Removalists are moving boxes of possessions for customers. The company has three different size boxes; small, medium and large. From the first location they collect 7 small, 2 medium and 2 large boxes and from the second location they collect 5 small boxes, 5 medium and 4 large boxes.

Use appropriate variables to represent the volume of different size boxes, determine an expression to represent the volume collected from each location. From your knowledge of like terms, add the two loads together to obtain a total volume.

Topic 3: Multiplying and dividing terms

When multiplying terms, it may be useful to insert multiplication sign as a reminder. The numbers and variables can then be rearranged. The numbers can then be multiplied as normal and the multiplication signs removed from the variables. With practise, some of these steps can be missed out.

Example:

Question	Solution
$2a \times 4b$	$2a \times 4b$ $= 2 \times a \times 4 \times b$ $= 2 \times 4 \times a \times b$ $= 8ab$
$3 \times 3a \times 4a$	$3 \times 3a \times 4a$ $= 3 \times 3 \times a \times 4 \times a$ $= 3 \times 3 \times 4 \times a \times a$ $= 36a^2$
$^-4ej \times 5fg \times 2h$	$^-4ej \times 5fg \times 2h$ $= {}^-4 \times e \times j \times 5 \times f \times g \times 2 \times h$ $= {}^-4 \times 5 \times 2 \times e \times f \times g \times h \times j$ $= {}^-40efghj$
Composite type $3a(2ab - 5ab)$	$3a(2ab - 5ab)$ $= 3a({}^-3ab)$ $= 3 \times {}^-3 \times a \times a \times b$ $= {}^-9a^2b$

When dividing terms, write the division as a fraction then simplify just like you would a fraction containing numbers.

Question	Solution
$16a\div 4$	$16a\div 4$ $=\frac{\cancel{16}^4\times a}{\cancel{4}^1}$ $=4a$
$3ab\div a$	$3ab\div a$ $=\frac{3\times \cancel{a}^1\times b}{\cancel{a}^1}$ $=\frac{3\times b}{1}=3b$
$^-45ef\div 9fg$	$^-45ef\div 9fg$ $=\frac{^-\cancel{45}^5\times e\times \cancel{f}^1}{\cancel{9}^1\times \cancel{f}^1\times g}$ $=\frac{^-5e}{g}$
$15ab\div 20b^2$	$15ab\div 20b^2$ $=\frac{15\times a\times b}{20\times b^2}$ $=\frac{\cancel{15}^3\times a\times \cancel{b}^1}{\cancel{20}^4\times b\times \cancel{b}^1}$ $=\frac{3a}{4b}$ or $15ab\div 20b^2$ $=\frac{15\times a\times b}{20\times b^2}$ $=\frac{\cancel{15}^3\times a\times \cancel{b}^{1-2}}{\cancel{20}^4}$ $=\frac{3ab^{-1}}{4}$ $=\frac{3a}{4b}$
$^-9a^2bc^3\div {}^-12ab^4c$	$^-9a^2bc^3\div {}^-12ab^4c$ $=\frac{^-\cancel{9}^3a^2bc^3}{^-\cancel{12}^4ab^4c}$ $=\frac{3\times a\times \cancel{a}^1\times \cancel{b}^1\times c\times c\times \cancel{c}^1}{4\times \cancel{a}^1\times b\times b\times b\times \cancel{b}^1\times \cancel{c}^1}$ $=\frac{3\times a\times c\times c}{4\times b\times b\times b}$ $=\frac{3ac^2}{4b^3}$ or $^-9a^2bc^3\div {}^-12ab^4c$ $=\frac{^-\cancel{9}^3a^2bc^3}{^-\cancel{12}^4ab^4c}$ $=\frac{3\times a^{2-1}\times b^{1-4}\times c^{3-1}}{4}$ $=\frac{3\times a^1\times b^{-3}\times c^2}{4}$ $=\frac{3ac^2}{4b^3}$

Composite question $3e \times 9f \div 15$	$3e \times 9f \div 15$ $= \dfrac{\cancel{27}^{9}ef}{\cancel{15}^{5}}$ $= \dfrac{9ef}{5}$

Video 'Multiplying and dividing terms'

Activity

1. Simplify the following.

(a)	$5\times 3a$	(b)	$8c\times 2a$	(c)	$11f\times 8g$
(d)	$^{-}3a\times 4c$	(e)	$3a\times 8a$	(f)	$2\times 4x\times 3y$
(g)	$4e\times 5ef$	(h)	$6k\times 3\times 2m$	(i)	$^{-}3a\times 4b\times {}^{-}5c$
(j)	$\frac{1}{2}p\times 8q$	(k)	$^{-}2ab\times 3bc\times {}^{-}ad$	(l)	$abc\times 4a$

2. Simplify the following.

(a)	$\frac{6a}{3}$	(b)	$\frac{12b}{b}$	(c)	$\frac{9g}{3g}$
(d)	$15x\div 3y$	(e)	$y\div 3y$	(f)	$30gh\div 5h$
(g)	$^{-}6wx\div 36wy$	(h)	$^{-}25p^{2}\div {}^{-}5p$	(i)	$3pq\div q^{2}$
(j)	$4e\div 16ef$	(k)	$^{-}ab^{2}c\div 2b$	(l)	$12ab^{2}c\div 6a^{2}b$

3. Simplify the following.

(a)	$3a\times 6b\div 2$	(b)	$3abc\times 5b\div 15ac$	(c)	$^{-}b\times ac\div a^{2}$

Topic 4: Expanding brackets

There are two major ideas in Algebra called Expanding and Factorising. Expanding is when the distributive law is used to remove brackets, taking a product and writing this as a sum. Factorising is the opposite of expanding, where a sum is written as a product.

Product Sum

$$3(a+b) = 3a+3b$$

Expanding

Sum Product

$$3a+3b = 3(a+b)$$

Factorising

To expand brackets, everything in the brackets must be multiplied by the term outside the brackets. To help in this process, one of two methods is used. Remember that when no operation is shown, multiplication is implied.

Example:

Method 1:	Method 2:
$3(a+b)$ $=3\times(a+b)$ $=3\times a+3\times b$ $=3a+3b$	$3(a+b)$ $=3\times a+3\times b$ $=3a+3b$
$7(a+4)$ $=7\times(a+4)$ $=7\times a+7\times 4$ $=7a+28$	$7(a+4)$ $=7\times a+7\times 4$ $=7a+28$
$a(b-7)$ $=a\times(b-7)$ $=a\times b-a\times 7$ $=ab-7a$	$a(b-7)$ $=a\times b-a\times 7$ $=ab-7a$
$3(4-3n)$ $=3\times(4-3n)$ $=3\times 4-3\times 3n$ $=12-9n$	$3(4-3n)$ $=3\times 4-3\times 3n$ $=12-9n$
$^-7a(a-5b)$ $=\ ^-7\times a\times(a-5b)$ $=\ ^-7\times a\times a-\ ^-7\times a\times 5b$ $=\ ^-7a^2+35ab$	$^-7a(a-5b)$ $=\ ^-7\times a\times a-\ ^-7\times a\times 5b$ $=\ ^-7a^2+35ab$
$^-3p(^-5p+4q)$ $=\ ^-3\times p\times(^-5p+4q)$ $=\ ^-3\times p\times\ ^-5p+\ ^-3\times p\times 4q$ $=15p^2-12pq$	$^-3p(^-5p+4q)$ $=\ ^-3\times p\times\ ^-5p+\ ^-3\times p\times 4q$ $=15p^2-12pq$

Expanding brackets and collecting like terms

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Expanding
$3(2a+5b)+4(a-1)$
$=6a+15b+4a-4$
Collecting like terms
$=10a+15b-4$

Expanding
$3a(b-3)-5(2ab+7)$
$=3ab-9a-10ab-35$
Collecting like terms
$= {}^{-}7ab-9a-35$

Expanding
$2a(b-3c)+5ab+6ac-12$
$=2ab-6ac+5ab+6ac-12$
Collecting like terms
$=7ab-12$

Video ‘Expanding brackets’

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Activity

1. Expand the following expressions:

(a)	$4(a-8)$	(b)	$3(10-r)$
(c)	$a(h+3)$	(d)	$b(3b-5)$
(e)	$2s(t-1)$	(f)	$n(n-m)$
(g)	$6(3-2a)$	(h)	$3m(n-4)$
(i)	$2d(3d-e)$	(j)	$^{-}4(4-3m)$
(k)	$4a(a+b-3c)$	(l)	$3(^{-}s+2t)$
(m)	$^{-}a(m-n)$	(n)	$5ij(j-5)$

2. Simplify the following by expanding the brackets and collecting like terms

(a)	$3(2a+b)+7a$	(b)	$5(a-3b)-a+2$
(c)	$6x-2(x-3y)$	(d)	$5xy+6x(y-3)+2x$
(e)	$4(2x-y)+3(x-2y)$	(f)	$4d(e+4)+5de-10d+9$
(g)	$^{-}2x(f+2g)+3(5g-f)$	(h)	$^{-}2h(2i-j)-3i(2h+j)$
(i)	$2(2x-3y+z)-3x(2-y)$	(j)	$7x(x-4)-x(7-3x)$

Topic 5: Factorising

Factorising is the opposite of expanding, where a sum or difference of terms is written as a product. In order to factorise, the terms must contain a common factor. The result is inserting brackets, making it the opposite of expanding.

$$\overbrace{3(a+b)}^{\text{Product}} = \overbrace{3a+3b}^{\text{Sum}}$$

Expanding

$$\overbrace{3a+3b}^{\text{Sum}} = \overbrace{3(a+b)}^{\text{Product}}$$

Factorising

Factorise $2a+6$

The first step is to look at the two terms and identify the **highest common factor**. The highest common factor can be a number, a variable or both.

In this case the highest common factor is 2. Write each term with 2 as a factor.

So far

$$\begin{aligned}&2a+6\\&=2\times a+2\times 3\end{aligned}$$

Put the common factor outside the brackets

Now it becomes

$$\begin{aligned}&2a+6\\&=2\times a+2\times 3\\&=2\times(a+3)\end{aligned}$$

Remove unnecessary multiplication signs
The solution is

$$\begin{aligned}&2a+6\\&=2\times a+2\times 3\\&=2\times(a+3)\\&=2(a+3)\end{aligned}$$

Check your answer by expanding the solution:

$$2(a+3) = 2 \times a + 2 \times 3 = 2a + 6,$$

gives the question, so there is some certainty that the solution is correct. With experience, some of the steps may be omitted.

Examples

Factorise	H.C.F.	Solution
$3f - 21$	3	$3f - 21$ $= 3 \times f - 3 \times 7$ $= 3 \times (f - 7)$ $= 3(f - 7)$
$2ab - a$	a	$2ab - a$ $= a \times 2b - a \times 1$ $= a \times (2b - 1)$ $= a(2b - 1)$
$3xy - 6xz$	$3x$	$3xy - 6xz$ $= 3x \times y - 3x \times 2z$ $= 3x \times (y - 2z)$ $= 3x(y - 2z)$
$x^2 - 4x$	x	$x^2 - 4x$ $= x \times x - x \times 4$ $= x \times (x - 4)$ $= x(x - 4)$
$ab^2c + 3ab$	ab	$ab^2c + 3ab$ $= ab \times bc + ab \times 3$ $= ab \times (bc + 3)$ $= ab(bc + 3)$
$^{-}25x^2 - 10x$ Hint: take out a negative c.f. if both terms are negative	$^{-}5x$	$^{-}25x^2 - 10x$ $= {^{-}5x} \times 5x + {^{-}5x} \times 2$ $= {^{-}5x} \times (5x + 2)$ $= {^{-}5x}(5x + 2)$

Video ‘Factorising’

Activity

1. What is the HCF of the terms 12 and 36

   (a) 3 (b) 6 (c) 12 (d) a

2. What is the HCF of the terms $^{-}3a$ and $^{-}15$

   (a) 3 (b) 5 (c) $^{-}3$ (d) $^{-}5$

3. What is the HCF of the terms $4a$ and $16a$

   (a) 4 (b) 16 (c) a (d) $4a$

4. What is the HCF of the terms $5x^2$ and $45x$

   (a) 5 (b) x (c) $5x$ (d) $5x^2$

5. Complete the following by filling in the brackets

   (a) $2p - 10 = 2(p - \quad)$

   (b) $18pq + 12p = 6p(\quad + \quad)$

   (c) $36 - 48pq = 12(\qquad)$

   (d) $^{-}4xy - 44y = {^{-}4y}(\qquad)$

   (e) $^{-}a + 1 = -(\qquad)$

6. Factorise the following expressions.

(a)	$5r - 35$	(b)	$18p - 2$
(c)	$ab - 5b$	(d)	$3gh - 9g$
(e)	$27cd + 81d$	(f)	$^{-}ax + a$
(g)	$6abc + 9c$	(h)	$4pq - 9pr$
(i)	$x^2 + 5x$	(j)	$4pq^2 - 10pq$

7. Collect like terms and then factorise

   (a) $5xy + 7x - xy$

   (b) $10a - 3b - a$

   (c) $5pq + pr - 3pq$

   (d) $4a^2 + 5a - 8 - a^2 + 4a$

Equations

1. Solve the equations below for the given variable

(a) $7x = 49$ or $7x = 49$

(b) $x + 5 = 2$ or $x + 5 = 2$

(c) $\frac{w}{5} = {}^{-}3$ or $\frac{w}{5} = {}^{-}3$

(d) $6 + k = 4$ or $6 + k = 4$

(e) $3x - 7 = 11$ $3x - 7 = 11$

(f) $p - 1.5 = 4.2$ or $p - 1.5 = 4.2$

(g) ${}^{-}3h = 21$ or ${}^{-}3h = 21$

(h) $\frac{g}{5} = \frac{2}{3}$ or $\frac{g}{5} = \frac{2}{3}$

(i) $5 - x = 7$ $5 - x = 7$ or $5 - x = 7$ $5 - x = 7$

(j) $6 = 2t - 7$ or $6 = 2t - 7$

t

(k) $\dfrac{x}{3} + 2 = 5$ or $\dfrac{x}{3} + 2 = 5$

(l) $\dfrac{x-4}{2} = 7$ or $\dfrac{x-4}{2} = 7$

(m) $\dfrac{4}{x} = \dfrac{5}{7}$ or $\dfrac{4}{x} = \dfrac{5}{7}$

(n) $^{-}x + 1 = 6$ or $^{-}x + 1 = 6$

x x

(o) $^{-}4 + 3x = 15$ or $^{-}4 + 3x = 15$

x x

(p) $\frac{3r}{4}-5=1$ or $\frac{3r}{4}-5=1$

(q) $\frac{5-2x}{3}=2$ or $\frac{5-2x}{3}=2$

(r) $\frac{2x-5}{4}={}^{-}7$ $\frac{2x-5}{4}={}^{-}7$

2. Solve the equations below for t.

(a) $7(t-1)=9$ or $7(t-1)=9$

(b) $2t + a = 2$ or $2t + a = 2$

(c) $\frac{w}{t} = {}^{-}3$ or $\frac{w}{t} = {}^{-}3$

(d) $3(2t - a) = 4$ or $3(2t - a) = 4$

(e) $3t - x + y = {}^{-}7$ or $3t - x + y = {}^{-}7$

(f) $4 - 3t = 7$ or $4 - 3t = 7$

t t

3. Solve the equations below for the variable used.

(a) $7t-1=5t+5$ or $7t-1=5t+5$

(b) $3x+11=3-x$ or $3x+11=3-x$

(c) $15-p=2p$ or $15-p=2p$

(d) $2(a-3)=4a-6$ or $2(a-3)=4a-6$

(e) $3(2s-7)=2(3s-1)$ or $3(2s-7)=2(3s-1)$

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(f) $5(y-2)=3(4-y)$ or $5(y-2)=3(4-y)$

(g) $0.6(3y-1)=0.4y-1$ or $0.6(3y-1)=0.4y-1$

(h) $4-7a=3(1-3a)$ or $4-7a=3(1-3a)$

(i) $2-(3x+9)=4(5-3x)$ or $2-(3x+9)=4(5-3x)$

4. Transpose the following equations for the subject given in the brackets.

(a) $P = \dfrac{W}{t}$ (W) or $P = \dfrac{W}{t}$ (W)

(b) $P = \dfrac{W}{t}$ (t) or $P = \dfrac{W}{t}$ (t)

(c) $A = l \times \quad b \quad (l)$ or $A = l \times b \quad (l)$

(d) $C = Vq + F \quad (q)$ or $C = Vq + F \quad (q)$

$=$

(e) $V = \dfrac{1}{3}\pi r^2 h \quad (h)$ or $V = \dfrac{1}{3}\pi r^2 h \quad (h)$

(f) $V = \frac{1}{3}\pi r^2 h \quad (r)$ or $V = \frac{1}{3}\pi r^2 h \quad (r)$

(g) $R = \frac{l}{\mu a} \quad (l)$ or $R = \frac{l}{\mu a} \quad (l)$

(h) $R = \frac{l}{\mu a} \quad (a)$ or $R = \frac{l}{\mu a} \quad (a)$

(i) $A = \frac{1}{2}bh \quad (b)$ or $A = \frac{1}{2}bh \quad (b)$

(j) $C = 2\pi r \quad (r)$ or $C = 2\pi r \quad (r)$

(k) $c^2 = a^2 + b^2 \quad (a)$ or $c^2 = a^2 + b^2 \quad (a)$

(l) $PV = nRT \quad (P)$ or $PV = nRT \quad (P)$

(m) $PV = nRT \quad (n)$ or $PV = nRT \quad (n)$

(n) $Q_1 = P(Q_2 - Q_1) \quad (P)$ or $Q_1 = P(Q_2 - Q_1) \quad (P)$

(o) $Q_1 = P(Q_2 - Q_1) \quad (Q_2)$ or $Q_1 = P(Q_2 - Q_1) \quad (Q_2)$

(p) $Q_1 = P(Q_2 - Q_1) \quad (Q_1)$ or $Q_1 = P(Q_2 - Q_1) \quad (Q_1)$

(q) $R_2 = R_1(1+\alpha t) \quad (t)$ or $R_2 = R_1(1+\alpha t) \quad (t)$

(r) $E = mc^2 \quad (c)$ or $E = mc^2 \quad (c)$

(s) $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v} \quad (v)$ or $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v} \quad (v)$

(t) $T = mg + m\omega^2 r \quad (r)$ or $T = mg + m\omega^2 r \quad (r)$

(u) $P = \dfrac{V_1(V_2 - V_1)}{gJ} \quad (V_2)$ or $P = \dfrac{V_1(V_2 - V_1)}{gJ} \quad (V_2)$

(v) $Z = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2} \quad (\omega)$ or $Z = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2} \quad (\omega)$

$$\frac{1}{\omega}$$

2

Answers to activity questions

Check your skills

1. If a=7, b=3, c=0.4 and d=$^{-}6$, calculate the value of:

(a) $ac-b$
$=7\times 0.4-3$
$=2.8-3$
$=^{-}0.2$

(b) $d^2-\dfrac{ab}{c}$
$=\left(^{-}6\right)^2-\dfrac{7\times 3}{0.4}$
$=36-\dfrac{21}{0.4}$
$=36-52.5$
$=^{-}16.5$

2. (a) Simplify
$3ab+7c+ab-5c$
$=4ab+2c$

(b) Simplify
$2ab^2-3a^2b-b^2a$
$=2ab^2-3a^2b-ab^2$
$=ab^2-3a^2b$

3. (a) Simplify
$3ab\times 6bc\times {}^{-}2d^2$
$=3\times 6\times {}^{-}2\times ab^2cd^2$
$={}^{-}36ab^2cd^2$

(b) Simplify
$4abc\div 8abd$
$=\dfrac{\cancel{4}^1\cancel{a}^1\cancel{b}^1c}{\cancel{8}^2\cancel{a}^1\cancel{b}^1d}$
$=\dfrac{c}{2d}$

4. Expand and collect like terms
$3x(a-5)+4x$
$=3ax-15x+4x$
$=3ax-11x$

5. Factorise the expression
$5ab-15ac$
$=5\times a\times b-5\times a\times 3\times c$
$=5a(b-3c)$

6. (a) Solve the equation

(b) Make B the subject in

$$\frac{4c-7}{7}=3$$
$$4c-7=3\times 7$$
$$4c-7=21$$
$$4c=21+7$$
$$4c=28$$
$$c=\frac{28}{4}$$
$$c=7$$

$$W=\frac{B^2}{2\mu}$$
$$2\mu W=B^2$$
$$\sqrt{2\mu W}=B$$
$$B=\sqrt{2\mu W}$$

Substitution into expressions, equations and formulae

1. Evaluate the following expressions when $m=5$ and $n=12$

(a)
$$m+n$$
$$=5+12$$
$$=17$$

(b)
$$2m-n$$
$$=2\times 5-12$$
$$=10-12$$
$$={}^{-}2$$

(c)
$$m^2-5m+6$$
$$=5^2-5\times 5+6$$
$$=25-25+6$$
$$=6$$

(d)
$$(n-m)^2+2n-m$$
$$=(12-5)^2+2\times 12-5$$
$$=7^2+24-5$$
$$=49+24-5$$
$$=68$$

2. Evaluate the following expressions when $a=2$, $b={}^{-}3$ and $c=\frac{1}{4}$

(a)
$$2a-3b+4c$$
$$=2\times 2-3\times {}^{-}3+4\times\frac{1}{4}$$
$$=4+9+1$$
$$=14$$

(b)
$$a\div c$$
$$=2\div\frac{1}{4}$$
$$=\frac{2}{1}\times\frac{4}{1}$$
$$=\frac{8}{1}$$
$$=8$$

(c)
$$ab^2-a^2c$$
$$=2\times\left({}^{-}3\right)^2-2^2\times\frac{1}{4}$$
$$=2\times 9-4\times\frac{1}{4}$$
$$=18-1$$
$$=17$$

(d)
$$\frac{b}{a}+c$$
$$=\frac{{}^{-}3}{2}+\frac{1}{4}$$
$$=\frac{{}^{-}6}{4}+\frac{1}{4}$$
$$=\frac{{}^{-}5}{4}\text{ or }{}^{-}1\frac{1}{4}$$

3.
$$s=\frac{d}{t}$$
$$s=\frac{250}{3}$$
$$s=83.\overline{3}\ km/hr$$

Units are an important part of the answer.

4. $P = mgh$
$P = 550 \times 9.8 \times 10$
$P = 53900\, kg\, m^2 / \sec^2$

Adding and subtracting like terms

1. Simplify these expressions by collecting like terms if possible

(a) $14p - 7p$
$= 7p$

(b) $^{-}j - 3j$
$= {}^{-}4j$

(c) $4ab + 3a - ab$
$= 3ab + 3a$

(d) $3x + y + 7x - 6y$
$= 10x - 5y$

(e) $4x^2 + 4x - 2 + x^2 - 8x + 5$
$= 5x^2 - 4x + 3$

(f) $4a + 1 - 9a + 11$
$= {}^{-}5a + 12$
or $12 - 5a$

(g) $^{-}q + 3q - 7 + 4q$
$= 6q - 7$

(h) $4de + 9ed - de + 4e$
$= 4de + 9de - de + 4e$
$= 12de + 4e$

(i) $9 - 2s + 2 + 2s$
$= 11$

(j) $7 - a + 5a^2 - a^3 - 2 + 2a - 3a^2 + a^3$
$= 5 + a + 2a^2$

2. Let s be the volume of the small box
Let m be the volume of the medium sized box
Let l be the volume of the large box

The Volume collected from the first location is
$V_1 = 7s + 2m + 2l$
The Volume collected from the second location is
$V_2 = 5s + 5m + 4l$
The total volume collected altogether is:
$V_T = V_1 + V_2$
$V_T = (7s + 2m + 2l) + (5s + 5m + 4l)$
$V_T = 12s + 7m + 6l$

Multiplying and dividing terms

1. Simplify the following

(a) $5 \times 3a$
$= 15a$

(b) $8c \times 2a$
$= 16ac$

(c) $11f \times 8g$
$= 88fg$

(d) $^{-}3a \times 4c$
$= {}^{-}12ac$

(e) $3a \times 8a$
$= 24a^2$

(f) $2 \times 4x \times 3y$
$= 24xy$

(g) $4e \times 5ef$
$= 20e^2 f$

(h) $6k \times 3 \times 2m$
$= 36km$

(i) ${}^{-}3a \times 4b \times {}^{-}5c$
$= 60abc$

(j) $\frac{1}{2} p \times 8q$
$= 4pq$

(k) ${}^{-}2ab \times 3bc \times {}^{-}ad$
$= 6a^2b^2cd$

(l) $abc \times 4a$
$= 4a^2bc$

2. Simplify the following.

(a) $\frac{6a}{3}$
$= \frac{\cancel{6}^2 a}{\cancel{3}^1}$
$= 2a$

(b) $\frac{12b}{b}$
$= \frac{12\cancel{b}^1}{\cancel{b}^1}$
$= 12$

(c) $\frac{9g}{3g}$
$= \frac{\cancel{9}^3 \cancel{g}^1}{\cancel{3}^1 \cancel{g}^1}$
$= 3$

(d) $15x \div 3y$
$= \frac{\cancel{15}^5 x}{\cancel{3}^1 y}$
$= \frac{5x}{y}$

(e) $y \div 3y$
$= \frac{\cancel{y}^1}{3\cancel{y}^1}$
$= \frac{1}{3}$

(f) $30gh \div 5h$
$= \frac{\cancel{30}^6 g\cancel{h}^1}{\cancel{5}^1 \cancel{h}^1}$
$= 6g$

(g) ${}^{-}6wx \div 36wy$
$= \frac{{}^{-}\cancel{6}^1 \cancel{w}^1 x}{\cancel{36}^6 \cancel{w}^1 y}$
$= \frac{{}^{-}x}{6y}$

(h) ${}^{-}25p^2 \div {}^{-}5p$
$= \frac{{}^{-}\cancel{25}^5 p^2}{{}^{-}\cancel{5}^1 p^1}$
$= 5p^{2-1}$
$= 5p^1$ or $5p$

(i) $3pq \div q^2$
$= \frac{3p\cancel{q}^1}{q \times \cancel{q}^1}$
$= \frac{3p}{q}$

(j) $4e \div 16ef$
$= \frac{\cancel{4}^1 \cancel{e}^1}{\cancel{16}^4 \cancel{e}^1 f}$
$= \frac{1}{4f}$

(k) ${}^{-}ab^2c \div 2b$
$= \frac{{}^{-}ab^2c}{2b^1}$
$= \frac{{}^{-}ab^{2-1}c}{2}$
$= \frac{{}^{-}abc}{2}$

(l) $12ab^2c \div 6a^2b$
$= \frac{\cancel{12}^2 \cancel{a}^1 b\cancel{b}^1 c}{\cancel{6}^1 \cancel{a}^1 a\cancel{b}^1}$
$= \frac{2bc}{a}$

3. Simplify the following.

(a) $3a \times 6b \div 2$
$= \frac{3a \times 6b}{2}$
$= \frac{\cancel{18}^9 ab}{\cancel{2}^1}$
$= 9ab$

(b) $3abc \times 5b \div 15ac$
$= \frac{3abc \times 5b}{15ac}$
$= \frac{\cancel{15}^1 \cancel{a}^1 b^2 \cancel{c}^1}{\cancel{15}^1 \cancel{a}^1 \cancel{c}^1}$
$= b^2$

(c) ${}^{-}b \times ac \div a^2$
$= \frac{{}^{-}b \times \cancel{a}^1 c}{a \times \cancel{a}^1}$
$= \frac{{}^{-}bc}{a}$

Expanding brackets

1. Expand the following expressions:

(a) $4(a-8)$
$=4a-32$

(b) $3(10-r)$
$=30-3r$

(c) $a(h+3)$
$=ah+3a$

(d) $b(3b-5)$
$=3b^2-5b$

(e) $2s(t-1)$
$=2st-2s$

(f) $n(n-m)$
$=n^2-mn$

(g) $6(3-2a)$
$=18-12a$

(h) $3m(n-4)$
$=3mn-12m$

(i) $2d(3d-e)$
$=6d^2-2de$

(j) $^{-}4(4-3m)$
$= {}^{-}16+12m$
or $12m-16$

(k) $4a(a+b-3c)$
$=4a^2+4ab-12ac$

(l) $3({}^{-}s+2t)$
$= {}^{-}3s+6t$
or $6t-3s$

(m) $^{-}a(m-n)$
$= {}^{-}am+an$
or $an-am$

(n) $5ij(j-5)$
$=5ij^2-25ij$

2. Simplify the following by expanding the brackets and collecting like terms

(a) $3(2a+b)+7a$
$=6a+3b+7a$
$=13a+3b$

(b) $5(a-3b)-a+2$
$=5a-15b-a+2$
$=4a-15b+2$

(c) $6x-2(x-3y)$
$=6x-2x+6y$
$=4x+6y$

(d) $5xy+6x(y-3)+2x$
$=5xy+6xy-18x+2x$
$=11xy-16x$

(e) $4(2x-y)+3(x-2y)$
$=8x-4y+3x-6y$
$=11x-10y$

(f) $4d(e+4)+5de-10d+9$
$=4de+16d+5de-10d+9$
$=9de+6d+9$

(g) $^{-}2x(f+2g)+3(5g-f)$
$= {}^{-}2fx-4gx+15g-3f$
or $15g-2fx-4gx-3f$

(h) $^{-}2h(2i-j)-3i(2h+j)$
$= {}^{-}4hi+2hj-6hi-3ij$
$= {}^{-}10hi+2hj-3ij$
or $2hj-10hi-3ij$

(i) $2(2x-3y+z)-3x(2-y)$
$=4x-6y+2z-6x+3xy$
$={}^{-}2x-6y+2z+3xy$
or $3xy-2x-6y+2z$

(j) $7x(x-4)-x(7-3x)$
$=7x^2-28x-7x+3x^2$
$=10x^2-35x$

Factorising

1. What is the HCF of the terms 12 and 36

(a) 3 (b) 6 $\boxed{\text{(c) } 12}$ (d) a

2. What is the HCF of the terms $^{-}3a$ and $^{-}15$

(a) 3 (b) 5 $\boxed{\text{(c) } {}^{-}3}$ (d) $^{-}5$

3. What is the HCF of the terms $4a$ and $16a$

(a) 4 (b) 16 (c) a $\boxed{\text{(d) } 4a}$

4. What is the HCF of the terms $5x^2$ and $45x$

(a) 5 (b) x $\boxed{\text{(c) } 5x}$ (d) $5x^2$

5. Complete the following by filling in the brackets

(a) $2p-10=2(p-\ 5\ )$

(b) $18pq+12p=6p(3q+\ 2)$

(c) $36-48pq=12(3-4pq\ )$

(d) $^{-}4xy-44y={}^{-}4y(x+11)$

(e) $^{-}a+1=-(a-1)$

6. Factorise the following expressions.

(a) $5r-35$
$=5(r-7)$

(b) $18p-2$
$=2(9p-1)$

(c) $ab-5b$
$=b(a-5)$

(d) $3gh-9g$
$=3g(h-3)$

(e) $27cd+81d$
$=27d(c+3)$

(f) $^{-}ax+a$
$={}^{-}a(x-1)$

(g) $6abc+9c$
$=3c(2ab+3)$

(h) $4pq-9pr$
$=p(4q-9r)$

(i) x^2+5x
$=x(x+5)$

(j) $4pq^2-10pq$
$=2pq(2q-5)$

7. Collect like terms and then factorise

(a) $5xy + 7x - xy$
$= 4xy + 7x$
$= x(4y + 7)$

(b) $10a - 3b - a$
$= 9a - 3b$
$= 3(3a - b)$

(c) $5pq + pr - 3pq$
$= 2pq + pr$
$= p(2q + r)$

(d) $4a^2 + 5a - 8 - a^2 + 4a$
$= 3a^2 + 9a - 8$
No common factor.

Equations

1. Solve the equations below for the given variable

(a)
$$7x = 49 \qquad \text{or} \qquad 7x = 49$$
$$\frac{\cancel{7}^1 x}{\cancel{7}^1} = \frac{49}{7} \qquad \qquad x = \frac{49}{7}$$
$$x = 7 \qquad \qquad x = 7$$

(b)
$$x + 5 = 2 \qquad \text{or} \qquad x + 5 = 2$$
$$x + 5 - 5 = 2 - 5 \qquad \qquad x = 2 - 5$$
$$x = {}^{-}3 \qquad \qquad x = {}^{-}3$$

(c)
$$\frac{w}{5} = {}^{-}3 \qquad \text{or} \qquad \frac{w}{5} = {}^{-}3$$
$$\frac{\cancel{5}^1 \times w}{\cancel{5}^1} = {}^{-}3 \times 5 \qquad \qquad w = {}^{-}3 \times 5$$
$$w = {}^{-}15 \qquad \qquad w = {}^{-}15$$

(d)
$$6 + k = 4 \qquad \text{or} \qquad 6 + k = 4$$
$$6 - 6 + k = 4 - 6 \qquad \qquad k = 4 - 6$$
$$k = {}^{-}2 \qquad \qquad k = {}^{-}2$$

(e)
$$3x - 7 = 11 \qquad \qquad 3x - 7 = 11$$
$$3x - 7 + 7 = 11 + 7 \qquad \qquad 3x = 11 + 7$$
$$3x = 18 \qquad \qquad 3x = 18$$
$$\frac{3x}{3} = \frac{18}{3} \qquad \qquad x = \frac{18}{3}$$
$$x = 6 \qquad \qquad x = 6$$

(f)
$$p - 1.5 = 4.2 \qquad \text{or} \qquad p - 1.5 = 4.2$$
$$p - 1.5 + 1.5 = 4.2 + 1.5 \qquad \qquad p = 4.2 + 1.5$$
$$p = 5.7 \qquad \qquad p = 5.7$$

(g)
$${}^{-}3h = 21 \qquad \text{or} \qquad {}^{-}3h = 21$$
$$\frac{\cancel{{}^{-}3}^1 h}{\cancel{{}^{-}3}^1} = \frac{21}{{}^{-}3} \qquad \qquad h = \frac{21}{{}^{-}3}$$
$$h = {}^{-}7 \qquad \qquad h = {}^{-}7$$

(h)
$$\frac{g}{5} = \frac{2}{3} \qquad \text{or} \qquad \frac{g}{5} = \frac{2}{3}$$
$$\frac{\cancel{5}^1 \times g}{\cancel{5}^1} = \frac{2 \times 5}{3} \qquad \qquad g = \frac{2}{3} \times 5$$
$$g = \frac{10}{3} = 3\frac{1}{3} \qquad \qquad g = \frac{10}{3} = 3\frac{1}{3}$$

(i)
$$\begin{aligned} 5 - x &= 7 \\ 5 - x + x &= 7 + x \\ 5 &= 7 + x \\ 5 - 7 &= 7 - 7 + x \\ {}^{-}2 &= x \end{aligned} \quad \text{or} \quad \begin{aligned} 5 - x &= 7 \\ 5 - 5 - x &= 7 - 5 \\ {}^{-}x &= 2 \\ x &= {}^{-}2 \end{aligned}$$

or

$$\begin{aligned} 5 - x &= 7 \\ {}^{-}x &= 7 - 5 \\ {}^{-}x &= 2 \\ x &= {}^{-}2 \end{aligned} \quad \text{or} \quad \begin{aligned} 5 - x &= 7 \\ 5 &= 7 + x \\ 5 - 7 &= x \\ x &= {}^{-}2 \end{aligned}$$

(j)

$$6 = 2t - 7$$
$$6 + 7 = 2t - 7 + 7$$
$$13 = 2t$$
$$\frac{13}{2} = \frac{\cancel{2}^1 t}{\cancel{2}^1}$$
$$6\frac{1}{2} = t$$

or

$$6 = 2t - 7$$
$$6 + 7 = 2t$$
$$13 = 2t$$
$$\frac{13}{2} = t$$
$$t = 6\frac{1}{2}$$

(k)

$$\frac{x}{3} + 2 = 5$$
$$\frac{x}{3} + 2 - 2 = 5 - 2$$
$$\frac{x}{3} = 3$$
$$\frac{\cancel{3}^1 \times x}{\cancel{3}^1} = 3 \times 3$$
$$x = 9$$

or

$$\frac{x}{3} + 2 = 5$$
$$\frac{x}{3} = 5 - 2$$
$$\frac{x}{3} = 3$$
$$x = 3 \times 3$$
$$x = 9$$

(l)

$$\frac{x-4}{2} = 7$$
$$\frac{\cancel{2}^1 \times (x-4)}{\cancel{2}^1} = 7 \times 2$$
$$x - 4 = 14$$
$$x - 4 + 4 = 14 + 4$$
$$x = 18$$

or

$$\frac{x-4}{2} = 7$$
$$x - 4 = 7 \times 2$$
$$x - 4 = 14$$
$$x = 14 + 4$$
$$x = 18$$

(m)

$$\frac{4}{x} = \frac{5}{7}$$
$$\frac{x}{4} = \frac{7}{5}$$
$$\frac{\cancel{4}^1 \times x}{\cancel{4}^1} = \frac{4 \times 7}{5}$$
$$x = \frac{28}{5} = 5\frac{3}{5} \text{ or } 5.6$$

or

$$\frac{4}{x} = \frac{5}{7}$$
$$\frac{x}{4} = \frac{7}{5}$$
$$x = \frac{7 \times 4}{5}$$
$$x = \frac{28}{5} = 5\frac{3}{5} \text{ or } 5.6$$

(n)

$${}^{-}x + 1 = 6$$
$${}^{-}x + 1 - 1 = 6 - 1$$
$${}^{-}x = 5$$
$$x = {}^{-}5$$

or

$${}^{-}x + 1 = 6$$
$${}^{-}x = 6 - 1$$
$${}^{-}x = 5$$
$$x = {}^{-}5$$

(o)

$${}^{-}4 + 3x = 15$$
$${}^{-}4 + 4 + 3x = 15 + 4$$
$$3x = 19$$
$$\frac{3x}{3} = \frac{19}{3}$$
$$x = 6\frac{1}{3}$$

or

$${}^{-}4 + 3x = 15$$
$$3x = 15 + 4$$
$$3x = 19$$
$$x = \frac{19}{3}$$
$$x = 6\frac{1}{3}$$

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(p)

$$\frac{3r}{4}-5=1$$
$$\frac{3r}{4}-5+5=1+5$$
$$\frac{3r}{4}=6$$
$$\frac{\cancel{4}^1\times 3r}{\cancel{4}^1}=6\times 4$$
$$3r=24$$
$$\frac{\cancel{3}^1 r}{\cancel{3}^1}=\frac{24}{3}$$
$$r=8$$

or

$$\frac{3r}{4}-5=1$$
$$\frac{3r}{4}=1+5$$
$$\frac{3r}{4}=6$$
$$3r=6\times 4$$
$$3r=24$$
$$r=\frac{24}{3}$$
$$r=8$$

(q)

$$\frac{5-2x}{3}=2$$
$$\frac{\cancel{3}^1\times(5-2x)}{\cancel{3}^1}=2\times 3$$
$$5-2x=6$$
$$5-5-2x=6-5$$
$$^{-}2x=1$$
$$\frac{\cancel{^{-}2}^1 x}{\cancel{^{-}2}^1}=\frac{1}{^{-}2}$$
$$x=-\frac{1}{2}$$

or

$$\frac{5-2x}{3}=2$$
$$5-2x=2\times 3$$
$$5-2x=6$$
$$^{-}2x=6-5$$
$$^{-}2x=1$$
$$x=\frac{1}{^{-}2}\text{ or }-\frac{1}{2}$$

(r)

$$\frac{2x-5}{4}={}^{-}7$$
$$\frac{\cancel{4}^1\times(2x-5)}{\cancel{4}^1}={}^{-}7\times 4$$
$$2x-5={}^{-}28$$
$$2x-5+5={}^{-}28+5$$
$$2x={}^{-}23$$
$$\frac{\cancel{2}^1 x}{\cancel{2}^1}=\frac{^{-}23}{2}$$
$$x={}^{-}11\frac{1}{2}$$

$$\frac{2x-5}{4}={}^{-}7$$
$$2x-5={}^{-}7\times 4$$
$$2x-5={}^{-}28$$
$$2x={}^{-}28+5$$
$$2x={}^{-}23$$
$$x=\frac{^{-}23}{2}$$
$$x={}^{-}11\frac{1}{2}$$

2. Solve the equations below for t.

(a)

$$7(t-1)=9$$
$$7t-7=9$$
$$7t-7+7=9+7$$
$$7t=16$$
$$\frac{\cancel{7}^1 t}{\cancel{7}^1}=\frac{16}{7}$$
$$t=2\frac{2}{7}$$

or

$$7(t-1)=9$$
$$7t-7=9$$
$$7t=9+7$$
$$7t=16$$
$$t=\frac{16}{7}=2\frac{2}{7}$$

(b)

$$\begin{aligned}2t+a&=2\\2t+a-a&=2-a\\2t&=2-a\\\frac{\cancel{2}^1t}{\cancel{2}^1}&=\frac{2-a}{2}\\t&=\frac{2-a}{2}\end{aligned}$$

or

$$\begin{aligned}2t+a&=2\\2t&=2-a\\t&=\frac{2-a}{2}\end{aligned}$$

(c)

$$\begin{aligned}\frac{w}{t}&={}^-3\\\frac{\cancel{t}^1\times w}{\cancel{t}^1}&={}^-3\times t\\w&={}^-3t\\-\frac{w}{3}&=t\end{aligned}$$

or

$$\begin{aligned}\frac{w}{t}&={}^-3\\w&={}^-3t\\-\frac{w}{3}&=t\end{aligned}$$

(d)

$$\begin{aligned}3(2t-a)&=4\\6t-3a&=4\\6t-3a+3a&=4+3a\\6t&=4+3a\\\frac{6t}{6}&=\frac{4+3a}{6}\\t&=\frac{4+3a}{6}\end{aligned}$$

or

$$\begin{aligned}3(2t-a)&=4\\6t-3a&=4\\6t&=4+3a\\t&=\frac{4+3a}{6}\end{aligned}$$

(e)

$$\begin{aligned}3t-x+y&={}^-7\\3t-x+y-y&={}^-7-y\\3t-x&={}^-7-y\\3t-x+x&={}^-7-y+x\\3t&={}^-7-y+x\\\frac{\cancel{3}^1t}{\cancel{3}^1}&=\frac{{}^-7-y+x}{3}\\t&=\frac{{}^-7-y+x}{3}\\t&=\frac{x-7-y}{3}\end{aligned}$$

or

$$\begin{aligned}3t-x+y&={}^-7\\3t-x&={}^-7-y\\3t&=x-7-y\\t&=\frac{x-7-y}{3}\end{aligned}$$

(f)

$$\begin{aligned}4-3t&=7\\4-4-3t&=7-4\\{}^-3t&=3\\\frac{{}^-\cancel{3}^1t}{{}^-\cancel{3}^1}&=\frac{3}{{}^-3}\\t&={}^-1\end{aligned}$$

or

$$\begin{aligned}4-3t&=7\\{}^-3t&=7-4\\{}^-3t&=3\\t&=\frac{3}{{}^-3}={}^-1\end{aligned}$$

3. Solve the equations below for the variable used.

(a)

$$7t-1=5t+5$$
$$7t-1+1=5t+5+1$$
$$7t=5t+6$$
$$7t-5t=5t-5t+6$$
$$2t=6$$
$$\frac{\cancel{2}^1 t}{\cancel{2}^1}=\frac{6}{2}$$
$$t=3$$

or

$$7t-1=5t+5$$
$$7t=5t+5+1$$
$$7t=5t+6$$
$$7t-5t=6$$
$$2t=6$$
$$t=\frac{6}{2}$$
$$t=3$$

(b)

$$3x+11=3-x$$
$$3x+11-11=3-11-x$$
$$3x={}^{-}8-x$$
$$3x+x={}^{-}8-x+x$$
$$4x={}^{-}8$$
$$\frac{\cancel{4}^1 x}{\cancel{4}^1}=\frac{{}^{-}8}{4}$$
$$x={}^{-}2$$

or

$$3x+11=3-x$$
$$3x=3-11-x$$
$$3x={}^{-}8-x$$
$$3x+x={}^{-}8$$
$$4x={}^{-}8$$
$$x=\frac{{}^{-}8}{4}$$
$$x={}^{-}2$$

(c)

$$15-p=2p$$
$$15-p+p=2p+p$$
$$15=3p$$
$$\frac{15}{3}=\frac{\cancel{3}^1 p}{\cancel{3}^1}$$
$$5=p$$

or

$$15-p=2p$$
$$15=2p+p$$
$$15=3p$$
$$\frac{15}{3}=p$$
$$5=p$$

(d)

$$2(a-3)=4a-6$$
$$2a-6=4a-6$$
$$2a-6+6=4a-6+6$$
$$2a=4a$$
$$2a-2a=4a-2a$$
$$0=2a$$
$$\frac{0}{2}=\frac{\cancel{2}^1 a}{\cancel{2}^1}$$
$$0=a$$

or

$$2(a-3)=4a-6$$
$$2a-6=4a-6$$
$$2a=4a-6+6$$
$$2a=4a$$
$$0=4a-2a$$
$$0=2a$$
$$\frac{0}{2}=a$$
$$0=a$$

(e)

$$3(2s-7)=2(3s-1)$$
$$6s-21=6s-2$$
$$6s-21+21=6s-2+21$$
$$6s=6s+19$$
$$6s-6s=6s-6s+19$$
$$0=19!$$

There is no solution

or

$$3(2s-7)=2(3s-1)$$
$$6s-21=6s-2$$
$$6s=6s-2+21$$
$$6s=6s+19$$
$$6s-6s=19$$
$$0=19!$$

No solution

(f)

$$
\begin{aligned}
5(y-2) &= 3(4-y)\\
5y-10 &= 12-3y\\
5y-10+10 &= 12+10-3y\\
5y &= 22-3y\\
5y+3y &= 22-3y+3y\\
8y &= 22\\
\frac{\cancel{8}^1 y}{\cancel{8}^1} &= \frac{\cancel{22}^{11}}{\cancel{8}^4}\\
y &= \frac{11}{4} = 2\frac{3}{4}
\end{aligned}
$$

or

$$
\begin{aligned}
5(y-2) &= 3(4-y)\\
5y-10 &= 12-3y\\
5y &= 12+10-3y\\
5y &= 22-3y\\
5y+3y &= 22\\
8y &= 22\\
y &= \frac{22}{8} = 2\frac{6}{8} = 2\frac{3}{4}
\end{aligned}
$$

(g)

$$
\begin{aligned}
0.6(3y-1) &= 0.4y-1\\
1.8y-0.6 &= 0.4y-1\\
1.8y-0.6+0.6 &= 0.4y-1+0.6\\
1.8y &= 0.4y-0.4\\
1.8y-0.4y &= 0.4y-0.4y-0.4\\
1.4y &= {}^-0.4\\
\frac{1.4y}{1.4} &= \frac{{}^-0.4}{1.4}\\
y &= \frac{{}^-0.4}{1.4} = \frac{{}^-4}{14} = \frac{{}^-2}{7}
\end{aligned}
$$

or

$$
\begin{aligned}
0.6(3y-1) &= 0.4y-1\\
1.8y-0.6 &= 0.4y-1\\
1.8y &= 0.4y-1+0.6\\
1.8y &= 0.4y-0.4\\
1.8y-0.4y &= {}^-0.4\\
1.4y &= {}^-0.4\\
y &= \frac{{}^-0.4}{1.4} = \frac{{}^-4}{14} = \frac{{}^-2}{7}
\end{aligned}
$$

(h)

$$
\begin{aligned}
4-7a &= 3(1-3a)\\
4-7a &= 3-9a\\
4-4-7a &= 3-4-9a\\
{}^-7a &= {}^-1-9a\\
{}^-7a+9a &= {}^-1-9a+9a\\
2a &= {}^-1\\
\frac{2a}{2} &= \frac{{}^-1}{2}\\
a &= \frac{{}^-1}{2}
\end{aligned}
$$

or

$$
\begin{aligned}
4-7a &= 3(1-3a)\\
4-7a &= 3-9a\\
{}^-7a &= 3-4-9a\\
{}^-7a &= {}^-1-9a\\
{}^-7a+9a &= {}^-1\\
2a &= {}^-1\\
a &= \frac{{}^-1}{2}
\end{aligned}
$$

(i)

$$
\begin{aligned}
2-(3x+9) &= 4(5-3x)\\
2-3x-9 &= 20-12x\\
{}^-3x-7 &= 20-12x\\
{}^-3x-7+7 &= 20+7-12x\\
{}^-3x &= 27-12x\\
{}^-3x+12x &= 27-12x+12x\\
9x &= 27\\
\frac{\cancel{9}^1 x}{\cancel{9}^1} &= \frac{27}{9}\\
x &= 3
\end{aligned}
$$

or

$$
\begin{aligned}
2-(3x+9) &= 4(5-3x)\\
2-3x-9 &= 20-12x\\
{}^-3x-7 &= 20-12x\\
{}^-3x &= 20+7-12x\\
{}^-3x &= 27-12x\\
{}^-3x+12x &= 27\\
9x &= 27\\
x &= \frac{27}{9}\\
x &= 3
\end{aligned}
$$

4. Transpose the following equations for the subject given in the brackets.

(a)

$$P=\frac{W}{t}\quad(W)$$
$$Pt=\frac{\cancel{t}^{1}W}{\cancel{t}^{1}}$$
$$Pt=W$$

or

$$P=\frac{W}{t}\quad(W)$$
$$Pt=W$$

(b)

$$P=\frac{W}{t}\quad(t)$$
$$\frac{P}{1}=\frac{W}{t}$$
$$\frac{1}{P}=\frac{t}{W}$$
$$\frac{1\times W}{P}=\frac{t\times \cancel{W}^{1}}{\cancel{W}^{1}}$$
$$\frac{W}{P}=t$$

or

$$P=\frac{W}{t}\quad(t)$$
$$Pt=W$$
$$t=\frac{W}{P}$$

(c)

$$A=l\times b\quad(l)$$
$$\frac{A}{b}=\frac{l\times \cancel{b}^{1}}{\cancel{b}^{1}}$$
$$\frac{A}{b}=l$$

or

$$A=l\times b\quad(l)$$
$$\frac{A}{b}=l$$

(d)

$$C=Vq+F\quad(q)$$
$$C-F=Vq+F-F$$
$$C-F=Vq$$
$$\frac{C-F}{V}=\frac{\cancel{V}^{1}q}{\cancel{V}^{1}}$$
$$\frac{C-F}{V}=q$$

or

$$C=Vq+F\quad(q)$$
$$C-F=Vq$$
$$\frac{C-F}{V}=q$$

(e)

$$V=\frac{1}{3}\pi r^{2}h\quad(h)$$
$$3\times V=3\times\frac{1}{3}\pi r^{2}h$$
$$3V=\pi r^{2}h$$
$$\frac{3V}{\pi r^{2}}=\frac{\cancel{\pi r^{2}}^{1}h}{\cancel{\pi r^{2}}^{1}}$$
$$\frac{3V}{\pi r^{2}}=h$$

or

$$V=\frac{1}{3}\pi r^{2}h\quad(h)$$
$$3V=\pi r^{2}h$$
$$\frac{3V}{\pi r^{2}}=h$$

(f)

$$V = \frac{1}{3}\pi r^2 h \quad (r)$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi h} = \frac{\cancel{\pi}^1 r^2 \cancel{h}^1}{\cancel{\pi}^1 \cancel{h}^1}$$
$$\frac{3V}{\pi h} = r^2$$
$$\sqrt{\frac{3V}{\pi h}} = \sqrt{r^2}$$
$$\sqrt{\frac{3V}{\pi h}} = r$$

or

$$V = \frac{1}{3}\pi r^2 h \quad (r)$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi h} = r^2$$
$$\sqrt{\frac{3V}{\pi h}} = r$$

(g)

$$R = \frac{l}{\mu a} \quad (l)$$
$$R \times \mu a = \frac{l \times \cancel{\mu a}^1}{\cancel{\mu a}^1}$$
$$R\mu a = l$$

or

$$R = \frac{l}{\mu a} \quad (l)$$
$$R\mu a = l$$

(h)

$$R = \frac{l}{\mu a} \quad (a)$$
$$R \times \mu a = \frac{l \times \cancel{\mu a}^1}{\cancel{\mu a}^1}$$
$$R\mu a = l$$
$$\frac{\cancel{R\mu}^1 a}{\cancel{R\mu}^1} = \frac{l}{R\mu}$$
$$a = \frac{l}{R\mu}$$

or

$$R = \frac{l}{\mu a} \quad (a)$$
$$R\mu a = l$$
$$a = \frac{l}{R\mu}$$

(i)

$$A = \frac{1}{2}bh \quad (b)$$
$$2 \times A = 2 \times \frac{1}{2}bh$$
$$2A = bh$$
$$\frac{2A}{h} = \frac{b\cancel{h}^1}{\cancel{h}^1}$$
$$\frac{2A}{h} = b$$

or

$$A = \frac{1}{2}bh \quad (b)$$
$$2A = bh$$
$$\frac{2A}{h} = b$$

(j)

$$C = 2\pi r \quad (r)$$
$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$
$$\frac{C}{2\pi} = r$$

or

$$C = 2\pi r \quad (r)$$
$$\frac{C}{2\pi} = r$$

(k)

$$c^2 = a^2 + b^2 \quad (a)$$
$$c^2 - b^2 = a^2 + b^2 - b^2$$
$$c^2 - b^2 = a^2$$
$$\sqrt{c^2 - b^2} = \sqrt{a^2}$$
$$\sqrt{c^2 - b^2} = a$$

or

$$c^2 = a^2 + b^2 \quad (a)$$
$$c^2 - b^2 = a^2$$
$$\sqrt{c^2 - b^2} = a$$

(l)

$$PV = nRT \quad (P)$$
$$\frac{P\cancel{V}^1}{\cancel{V}^1} = \frac{nRT}{V}$$
$$P = \frac{nRT}{V}$$

or

$$PV = nRT \quad (P)$$
$$P = \frac{nRT}{V}$$

(m)

$$PV = nRT \quad (n)$$
$$\frac{PV}{RT} = \frac{n\,\cancel{RT}^1}{\cancel{RT}^1}$$
$$\frac{PV}{RT} = n$$

or

$$PV = nRT \quad (n)$$
$$\frac{PV}{RT} = n$$

(n)

$$Q_1 = P(Q_2 - Q_1) \quad (P)$$
$$\frac{Q_1}{(Q_2 - Q_1)} = \frac{P\,\cancel{(Q_2 - Q_1)}^1}{\cancel{(Q_2 - Q_1)}^1}$$
$$\frac{Q_1}{(Q_2 - Q_1)} = P$$

or

$$Q_1 = P(Q_2 - Q_1) \quad (P)$$
$$\frac{Q_1}{(Q_2 - Q_1)} = P$$

(o)

$$Q_1 = P(Q_2 - Q_1) \quad (Q_2)$$
$$Q_1 = PQ_2 - PQ_1$$
$$Q_1 + PQ_1 = PQ_2 - PQ_1 + PQ_1$$
$$Q_1 + PQ_1 = PQ_2$$
$$\frac{Q_1 + PQ_1}{P} = \frac{PQ_2}{P}$$
$$\frac{Q_1 + PQ_1}{P} = Q_2$$

or

$$Q_1 = P(Q_2 - Q_1) \quad (Q_2)$$
$$Q_1 = PQ_2 - PQ_1$$
$$Q_1 + PQ_1 = PQ_2$$
$$\frac{Q_1 + PQ_1}{P} = Q_2$$

(p)

$$Q_1 = P(Q_2 - Q_1) \quad (Q_1)$$
$$Q_1 = PQ_2 - PQ_1$$
$$Q_1 + PQ_1 = PQ_2 - PQ_1 + PQ_1$$
$$Q_1 + PQ_1 = PQ_2$$
$$Q_1(1 + P) = PQ_2$$
$$\frac{Q_1(1 + P)}{(1 + P)} = \frac{PQ_2}{(1 + P)}$$
$$Q_1 = \frac{PQ_2}{(1 + P)}$$

or

$$Q_1 = P(Q_2 - Q_1) \quad (Q_1)$$
$$Q_1 = PQ_2 - PQ_1$$
$$Q_1 + PQ_1 = PQ_2$$
$$Q_1(1 + P) = PQ_2$$
$$Q_1 = \frac{PQ_2}{(1 + P)}$$

(q)

$$R_2 = R_1(1+\alpha t) \quad (t)$$
$$R_2 = R_1 + R_1\alpha t$$
$$R_2 - R_1 = R_1 - R_1 + R_1\alpha t$$
$$R_2 - R_1 = R_1\alpha t$$
$$\frac{R_2 - R_1}{R_1\alpha} = \frac{\cancel{R_1\alpha}^1 t}{\cancel{R_1\alpha}^1}$$
$$\frac{R_2 - R_1}{R_1\alpha} = t$$

or

$$R_2 = R_1(1+\alpha t) \quad (t)$$
$$R_2 = R_1 + R_1\alpha t$$
$$R_2 - R_1 = R_1\alpha t$$
$$\frac{R_2 - R_1}{R_1\alpha} = t$$

(r)

$$E = mc^2 \quad (c)$$
$$\frac{E}{m} = \frac{\cancel{m}^1 c^2}{\cancel{m}^1}$$
$$\frac{E}{m} = c^2$$
$$\sqrt{\frac{E}{m}} = \sqrt{c^2}$$
$$\sqrt{\frac{E}{m}} = c$$

or

$$E = mc^2 \quad (c)$$
$$\frac{E}{m} = c^2$$
$$\sqrt{\frac{E}{m}} = c$$

(s)

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad (v)$$
$$\frac{1}{f} - \frac{1}{u} = \frac{1}{u} - \frac{1}{u} + \frac{1}{v}$$
$$\frac{1}{f} - \frac{1}{u} = \frac{1}{v}$$
$$\frac{u-f}{fu} = \frac{1}{v}$$
$$\frac{v}{1} = \frac{fu}{u-f}$$
$$v = \frac{fu}{u-f}$$

or

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad (v)$$
$$\frac{1}{f} - \frac{1}{u} = +\frac{1}{v}$$
$$\frac{u-f}{fu} = \frac{1}{v}$$
$$v = \frac{fu}{u-f}$$

(t)

$$T = mg + m\omega^2 r \quad (r)$$
$$T - mg = mg - mg + m\omega^2 r$$
$$T - mg = m\omega^2 r$$
$$\frac{T-mg}{m\omega^2} = \frac{\cancel{m\omega^2}^1 r}{\cancel{m\omega^2}^1}$$
$$\frac{T-mg}{m\omega^2} = r$$

or

$$T = mg + m\omega^2 r \quad (r)$$
$$T - mg = m\omega^2 r$$
$$\frac{T-mg}{m\omega^2} = r$$

(u)

$$P = \frac{V_1(V_2 - V_1)}{gJ} \quad (V_2)$$

$$P = \frac{V_1V_2 - (V_1)^2}{gJ}$$

$$P \times gJ = \frac{V_1V_2 - (V_1)^2}{\cancel{gJ}^1} \times \frac{\cancel{gJ}^1}{1}$$

$$PgJ = V_1V_2 - (V_1)^2$$

$$PgJ + (V_1)^2 = V_1V_2 - (V_1)^2 + (V_1)^2$$

$$PgJ + (V_1)^2 = V_1V_2$$

$$\frac{PgJ + (V_1)^2}{V_1} = \frac{\cancel{V_1}^1 V_2}{\cancel{V_1}^1}$$

$$\frac{PgJ + (V_1)^2}{V_1} = V_2$$

or

$$P = \frac{V_1(V_2 - V_1)}{gJ} \quad (V_2)$$

$$P = \frac{V_1V_2 - (V_1)^2}{gJ}$$

$$PgJ = V_1V_2 - (V_1)^2$$

$$PgJ + (V_1)^2 = V_1V_2$$

$$\frac{PgJ + (V_1)^2}{V_1} = V_2$$

(v)

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \quad (\omega)$$

$$Z^2 = \left(\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}\right)^2$$

$$Z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2$$

$$Z^2 - R^2 = R^2 - R^2 + \left(\frac{1}{\omega C}\right)^2$$

$$Z^2 - R^2 = \left(\frac{1}{\omega C}\right)^2$$

$$\sqrt{Z^2 - R^2} = \sqrt{\left(\frac{1}{\omega C}\right)^2}$$

$$\sqrt{Z^2 - R^2} = \frac{1}{\omega C}$$

$$\omega \times \sqrt{Z^2 - R^2} = \frac{1 \times \cancel{\omega}^1}{\cancel{\omega}^1 C}$$

$$\omega\sqrt{Z^2 - R^2} = \frac{1}{C}$$

$$\frac{\omega \cancel{\sqrt{Z^2 - R^2}}^1}{\cancel{\sqrt{Z^2 - R^2}}^1} = \frac{1}{C\sqrt{Z^2 - R^2}}$$

$$\omega = \frac{1}{C\sqrt{Z^2 - R^2}}$$

or

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \quad (\omega)$$

$$Z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2$$

$$Z^2 - R^2 = \left(\frac{1}{\omega C}\right)^2$$

$$\sqrt{Z^2 - R^2} = \frac{1}{\omega C}$$

$$\omega\sqrt{Z^2 - R^2} = \frac{1}{C}$$

$$\omega = \frac{1}{C\sqrt{Z^2 - R^2}}$$

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